Quiz 8

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Use cylindrical coordinates to **setup** $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-2$ and $z=2$. (**DO NOT SOLVE**)

Solution:

$$\int_0^{2\pi}\int_0^4\int_{-2}^2 r^2\, dz\, dr\, d\theta$$

2. (2 points) Evaluate the integral $\iiint_S 3\rho^2\sin\phi\, d\rho\, d\theta\, d\phi$, where $S=\{(\rho,\theta,\phi): 0\le\rho\le\cos(\phi),\ 0\le\theta\le 2\pi,\ 0\le\phi\le\frac{\pi}{2}\}$.

Solution:

$$\begin{aligned}\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{\cos(\phi)} 3\rho^2\sin\phi\, d\rho\, d\theta\, d\phi &= \int_0^{\frac{\pi}{2}}\int_0^{2\pi}\rho^3\sin(\phi)\Big|_0^{cos(\phi)} d\theta d\phi\\ &= \int_0^{\frac{\pi}{2}}\int_0^{2\pi}(\cos(\phi))^3\sin(\phi)\, d\phi\\ &= 2\pi\int_0^{\frac{\pi}{2}}(\cos(\phi))^3\sin(\phi)\, d\phi\\ &= -2\pi\int_1^0 u^3\, du\\ &= \frac{\pi}{2}\end{aligned}$$