MA 213 Worksheet #1

Section 12.1

1 (a) *12.1.2* Sketch the points $(1, 5, 3)$, $(0, 2, -3)$, $(-3, 0, 2)$, and $(2, -2, -1)$ on a single set of coordinate axes.

(b) *12.1.3* Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz-plane? Which point lies in the xz-plane?

2 *12.1.7* Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

3 (a) *12.1.15* Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

(b) *12.1.45* Find an equation of the set of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$. Describe the set.

4 *12.1.35* Describe in words the region of $\mathbb{R}^3$ represented by

$$1 \leq x^2 + y^2 + z^2 \leq 5.$$

Draw a sketch of the region.

Additional Recommended Problems

5 *12.1.17* Show that the equation

$$x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$$

represents the equation of a sphere. Find its radius and center.

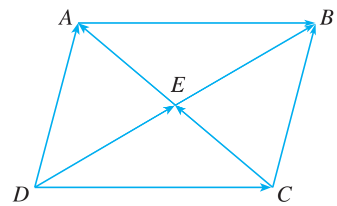
6 *12.1.40* Write inequalities to describe the solid that lies on or below the plane $z = 8$ and on or above the disc in the xy plane with center the origin and radius 2

7 *12.1.47* Find the distance between the spheres $x^2+y^2+z^2 = 4$ and $x^2+y^2+z^2 = 4x+4y+4z-11$.

MA 213 Worksheet #2

Section 12.2

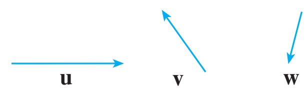
1 *12.2.3* Name all the equal vectors in the parallelogram shown.



2 *12.2.5* Copy the vectors in the figure and use them to draw the following vectors.

a $\mathbf{u}+\mathbf{v}$ **b** $\mathbf{u}+\mathbf{w}$

c $\mathbf{v}+\mathbf{u}+\mathbf{w}$ **d** $\mathbf{u}-\mathbf{v}-\mathbf{w}$



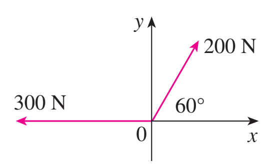
3 *12.2.13* For $A(0,3,1)$ and $B(2,3,-1)$, find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

4 *12.2.21* Find $\mathbf{a}+\mathbf{b}$, $4\mathbf{a}+2\mathbf{b}$, $|\mathbf{a}|$ and $|\mathbf{a}-\mathbf{b}|$ for $\mathbf{a}=4\mathbf{i}-3\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-4\mathbf{k}$.

5 *12.2.25* Find a unit vector that has the same direction as $8\mathbf{i}-\mathbf{j}+4\mathbf{k}$.

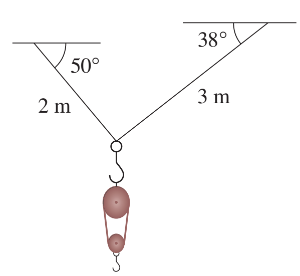
6 *12.2.29* If $\mathbf{v}$ lies in the first quadrant and makes an angle $\pi/3$ with the positive x-axis and $|\mathbf{v}|=4$, find $\mathbf{v}$ in component form.

7 *12.2.33* Find the magnitude of the resultant force and the angle it makes with the positive x-axis.



Additional Recommended Problems

8 *12.2.37* A block-and tackle pulley hoist is suspended in a warehouse by ropes of lengths 2 m and 3 m. The hoist weighs 350 N. The ropes, fastened at different heights, make angles of 50° and 38° with the horizontal. Find the tension in each rope and the magnitude of each tension.



9 *12.2.45* Let $\mathbf{a}=\langle 3,2\rangle$, $\mathbf{b}=\langle 2,-1\rangle$ and $\mathbf{c}=\langle 7,1\rangle$. Show, by means of sketch, that there are scalars s and t such that $\mathbf{c}=s\mathbf{a}+t\mathbf{b}$. Find the exact values of s and t.

10 *12.2.47* If $\mathbf{r}=\langle x,y,z\rangle$ and $\mathbf{r}_0=\langle x_0,y_0,z_0\rangle$, describe the set of all points (x,y,z) such that $|\mathbf{r}-\mathbf{r}_0|=1$.

MA 213 Worksheet #3

Section 12.3

1 Find $\mathbf{a} \cdot \mathbf{b}$ for the following descriptions of $\mathbf{a}$ and $\mathbf{b}$.

12.3.7 $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

12.3.9 $|\mathbf{a}| = 7$, $|\mathbf{b}| = 4$ the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/6$

2 *12.3.19* Find the angle between the vectors $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$.

3 *12.3.23* Determine whether the vectors $\mathbf{u} = 9\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = -6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ are orthogonal, parallel, or neither.

4 *12.3.25* Use vectors to decide whether the triangle with vertices $P(1, -3, -2)$, $Q(2, 0, -4)$, and $R(6, -2, -5)$ is right angled.

5 *12.3.41* Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

$$\mathbf{a} = \langle 4, 7, -4 \rangle, \quad \mathbf{b} = \langle 3, -1, 1 \rangle$$

6 *12.3.49* Find the work done by a force $\mathbf{F} = 8i - 6j + 9k$ that moves an object from the point $(0, 10, 8)$ to the point $(6, 12, 20)$ along a straight line. The distance is measured in meters and the force in newtons.

Additional Recommended Problems

7 *12.3.25* Find a unit vector that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

8 *12.3.30* Find the acute angle between the lines.

$$x + 2y = 7, \quad 5x - y = 2$$

9 *12.3.31* Find the acute angles between the curves at their points of intersection.

$$y = x^2, \quad y = x^3$$

10 *12.3.45* Show that the vector $\operatorname{orth}_{\mathbf{a}} \mathbf{b} = \mathbf{b} - \operatorname{proj}_{\mathbf{a}} \mathbf{b}$ is orthogonal to $\mathbf{a}$.

MA 213 Worksheet #4

Section 12.4

1 Find the cross product $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. Verify that $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

12.4.2 $\quad \mathbf{a} = \langle 4, 3, -2 \rangle, \qquad \mathbf{b} = \langle 2, -1, 1 \rangle$

12.4.5 $\quad \mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}, \quad \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

2 *12.4.20* Find two unit vectors orthogonal to both $\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$.

3 *12.4.29* For points $P(1, 0, 1)$, $Q(-2, 1, 3)$, and $R(4, 2, 5)$

(a) Find a nonzero vector orthogonal to the plane through the points P, Q, and R;

(b) Find the area of triangle PQR.

4 *12.4.34* Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Are these vectors coplanar?

5 *12.4.41* A wrench 30cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction $\langle 0, 3, -4 \rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100N·m of troque to the bolt.

6 *12.4.43* If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Additional Recommended Problems

7 *12.4.17* If $\mathbf{a} = \langle 2, -1, 3 \rangle$ and $\mathbf{b} = \langle 4, 2, 1 \rangle$, find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

8 *12.4.22* Explain why $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$ for all vectors $\mathbf{a}$ and $\mathbf{b}$ in V_3.

9 *12.4.37* Use the scalar triple product to verify that the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar.

10 *12.4.44* (a) Find all vectors $\mathbf{v}$ such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, -5 \rangle$$

(b) Explain why there is no vector $\mathbf{v}$ such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, 5 \rangle$$

MA 213 Worksheet #5

Section 12.5

1 (a) *12.5.3* Find the vector equation and the parametric equation of the line through the point $(2, 2.4, 3.5)$ and parallel to the vector $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$.

(b) *12.5.9* Find parametric equations and symmetric equations for the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$.

2 *12.5.31* Find an equation of the plane through points $(0, 1, 1), (1, 0, 1)$, and $(1, 1, 0)$.

3 *12.5.19* Determine whether the lines

$$L_1 : x = 3 + 2t, y = 4 - t, z = 1 + 3t$$
$$L_2 : x = 1 + 4s, y = 3 - 2s, z = 4 + 5s$$

are parallel, skew, or intersecting. If they intersect, find the point of intersection.

4 *12.5.30* Find an equation of the plane that contains the line $\langle x, y, z\rangle = \langle 1 + t, 2 - t, 4 - 3t\rangle$ and is parallel to the plane $5x + 2y + z = 1$.

5 *12.5.48* Where does the line through $(-3, 1, 0)$ and $(-1, 5, 6)$ intersect the plane $2x+y-z = -2$?

6 *12.5.53* Determine whether the planes $x + 2y - z = 2$ and $2x - 2y + z = 1$ are parallel, perpendicular, or neither. If neither, find the angle between them.

Additional Recommended Problems

7 *12.5.1* Determine whether each statement is true or false in 3D space. If true, explain why. If false, give a counterexample.

(a) Two lines parallel to a plane are parallel.

(b) Two planes perpendicular to a third plane are parallel.

(c) Two planes parallel to a third plane are parallel.

(d) Two lines perpendicular to a plane are parallel.

8 *12.5.21* Determine whether the lines

$$L_1 : \frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3} \quad \text{and} \quad L_2 : \frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$$

are parallel, skew, or intersecting. If they intersect, find the point of intersection.

9 *12.5.49* Find direction numbers for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

10 *12.5.61* Find an equation of the plane consisting of all the points that are equidistant from the points $(1, 0, -2)$ and $(3, 4, 0)$.

11 Assume that three points P, Q, and R are not collinear. Explain why the distance d between P and the line through Q and R is given by $d = \dfrac{\|\overrightarrow{QR} \times \overrightarrow{QP}\|}{\|\overrightarrow{QR}\|}$.

MA 213 Worksheet #6

Section 12.6

1 *12.6.1* (a) What does the equation $y = x^2$ represent as a curve in $\mathbb{R}^2$.
(b) What does it represent as a surface in $\mathbb{R}^3$
(c) What does the equation $z = y^2$ represent?

2 *12.6.5* Describe and sketch the surface $z = 1 - y^2$.

3 Use traces to sketch and identify the surfaces.
12.6.7 $xy = 1$
12.6.11 $x = y^2 + 4z^2$.

4 *12.6.21-28* **On back**

5 *12.6.37* Reduce the equation $x^2 - y^2 + z^2 - 4x - 2z = 0$ to one of the standard forms, classify the surface, and sketch it.

Additional Recommended Problems

6 *12.6.9*

(a) Find and identify the traces of the quadratic surface $x^2 + y^2 - z^2 = 1$.
(b) If we change the equation in part (a) to $x^2 - y^2 + z^2 = 1$, how is the graph affected?
(c) What if we change the equation in part (a) to $x^2 + y^2 + 2y - z^2 = 0$?

7 *12.6.35* Reduce the equation $x^2 + y^2 - 2x - 6y - z + 10 = 0$ to one of the standard forms, classify the surface, and sketch it.

8 *12.6.43* Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \le z \le 2$.

9 *12.6.52* Show that the curve of intersection of the surfaces $x^2 + 2y^2 - z^2 + 3x = 1$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$ lies in a plane.

21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choice.

21. $x^2 + 4y^2 + 9z^2 = 1$

22. $9x^2 + 4y^2 + z^2 = 1$

23. $x^2 - y^2 + z^2 = 1$

24. $-x^2 + y^2 - z^2 = 1$

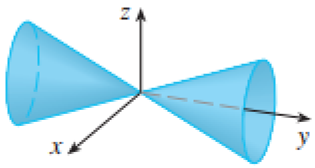
25. $y = 2x^2 + z^2$

26. $y^2 = x^2 + 2z^2$

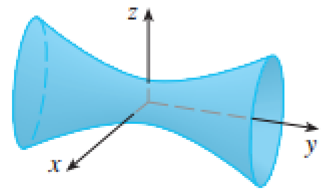
27. $x^2 + 2z^2 = 1$

28. $y = x^2 - z^2$

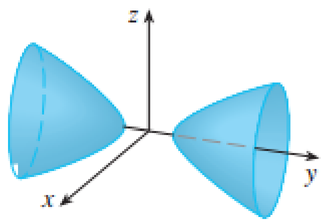
I



II



III



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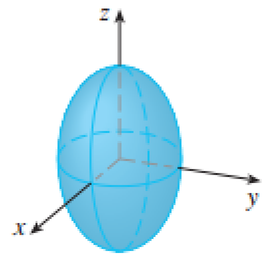

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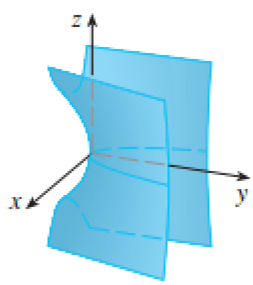
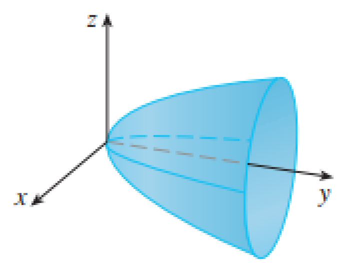
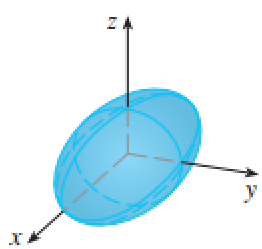


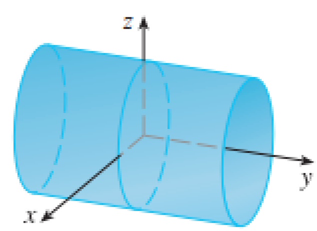
VI



VII



VIII



MA 213 Worksheet #7

Sections 13.1 and 13.2

1 *13.1.3* Find the limit: $\lim_{t\to 0}\left(e^{-3t}\mathbf{i}+\frac{t^2}{\sin^2 t}\mathbf{j}+\cos(2t)\mathbf{k}\right)$.

2 *13.1.17* Find a vector equation and parametric equations for the line segment that joins $P(2,0,0)$ to $Q(6,2,-2)$.

3 *13.1.43* Find a vector function that represents the curve of intersection of the cone $z=\sqrt{x^2+y^2}$ and the plane $z=1+y$.

4 *13.1.49* Suppose the trajectories of two particles are given by the vector functions $\mathbf{r}_1(t)=\langle t^2, 7t-12, t^2\rangle$ and $\mathbf{r}_2(t)=\langle 4t-3, t^2, 5t-6\rangle$ for $t\geq 0$. Do the particles collide?

5 *13.2.9* Find the derivative of the vector function $\mathbf{r}(t)=\langle\sqrt{t-2}, 3, 1/t^2\rangle$.

6 *13.2.23* Find the parametric equation for the tangent line to the curve given by: $x=t^2+1,\ \ y=4\sqrt{t}$ and $z=e^{t^2-t}$ at the point $(2,4,1)$.

7 *13.2.41* Find $\mathbf{r}(t)$ if $\mathbf{r}'(t)=2t\mathbf{i}+3t^2\mathbf{j}+\sqrt{t}\mathbf{k}$ and $\mathbf{r}(1)=\mathbf{i}+\mathbf{j}$.

Additional Recommended Problems

8 *13.1.7* Sketch the curve $\mathbf{r}(t)=\langle\sin t, t\rangle$. Indicate with an arrow the direction in which t increases.

9 *13.1.31* At what point does the curve $\mathbf{r}(t)=t\mathbf{i}+(2t-t^2)\mathbf{k}$ intersect the paraboloid $z=x^2+y^2$?

10 *13.2.33* The curves $\mathbf{r}_1(t)=\langle t, t^2, t^3\rangle$ and $\mathbf{r}_2(t)=\langle\sin t, \sin(2t), t\rangle$ intersect at the origin. Find their angle of intersection.

11 *13.2.35* Evaluate the integral: $\displaystyle\int_0^2\left(t\mathbf{i}-t^3\mathbf{j}+3t^5\mathbf{k}\right)dt$.

12 *13.2.49* Find $f'(2)$, where $f(t)=\mathbf{u}(t)\cdot\mathbf{v}(t)$, $\mathbf{u}(2)=\langle 1,2,-1\rangle$, $\mathbf{u}'(2)=\langle 3,0,4\rangle$ and $\mathbf{v}(t)=\langle t, t^2, t^3\rangle$.

MA 213 In-Class Work

Sections 13.3 and 13.4

1 Find the length of the following curves.

13.3.1 $\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t)\rangle \quad -5 \le t \le 5$

13.3.5 $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \quad 0 \le t \le 1$

2 *13.3.11* Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(6, 18, 36)$.

3 *13.3.13* Let $\mathbf{r}(t) = (5-t)\mathbf{i} + (4t-4)\mathbf{j} + 3t\mathbf{k}$.

a Find the arc length function for $\mathbf{r}(t)$ measured from the point $P = (4, 1, 3)$ in the direction of increasing t and then reparameterize the curve with respect to arc length starting from P.

b Find the point 4 units along $\mathbf{r}(t)$ (in the direction of increasing t) from P.

4 Find the velocity, acceleration and speed of a particle with the given position function. Sketch the path of the particle. Draw the velocity and acceleration vectors for the specified value of t.

13.4.3 $\mathbf{r}(t) = \langle -\frac{1}{2}t^2, t\rangle \quad t = 2$

13.4.7 $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 2\mathbf{k} \quad t = 1$

5 *13.4.15* Find the velocity and position vectors of a particle with acceleration vector $\mathbf{a}(t) = 2\mathbf{i} + 2t\mathbf{k}$, initial velocity $\mathbf{v}(0) = 3\mathbf{i} - \mathbf{j}$, and initial position $\mathbf{r}(0) = \mathbf{j} + \mathbf{k}$.

6 *13.4.23* A projectile is fired with an initial speed of 200 m/s and angle of elevation 60. Find

a the range of the projectile

b the maximum height reached

c the speed at impact

Additional Recommended Problems

7 *13.3.3* Find the length of the curve $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$ for $0 \le t \le 1$. *Hint:* $e^{2t} + 2 + e^{-2t}$ is a perfect square.

8 *13.3.15* Suppose you start at the point $(0, 0, 3)$ and move 5 units along the curve $x = 3\sin t$, $y = 4t$, $z = 3\cos t$ in the positive direction. Where are you now?

9 Find the unit tangent vector for the following curves.

13.3.17 $\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t)\rangle$

13.3.19 $\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^{-t}\rangle$

10 *13.4.25* A ball is thrown at an angle of $\pi/4$ to the ground. If the ball lands 90 m away, what was the initial speed of the ball?

MA 213 Worksheet #8

Section 14.1

1 *14.1.10* Let $F(x,y) = 1 + \sqrt{4 - y^2}$.
(a) Evaluate $F(3,1)$.
(b) Find and sketch the domain of F.
(c) Find the range of F.

2 *14.1.11* Find and describe the domain of $f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4 - x^2 - y^2 - z^2)$.

3 *14.1.49* Draw a contour map of $f(x,y) = ye^x$ showing several level curves.

4 *14.1.67* Describe the level surfaces of the function $f(x,y,z) = x + 3y + 5z$.

5 *14.1.61-66* **On back**

Additional Recommended Problems

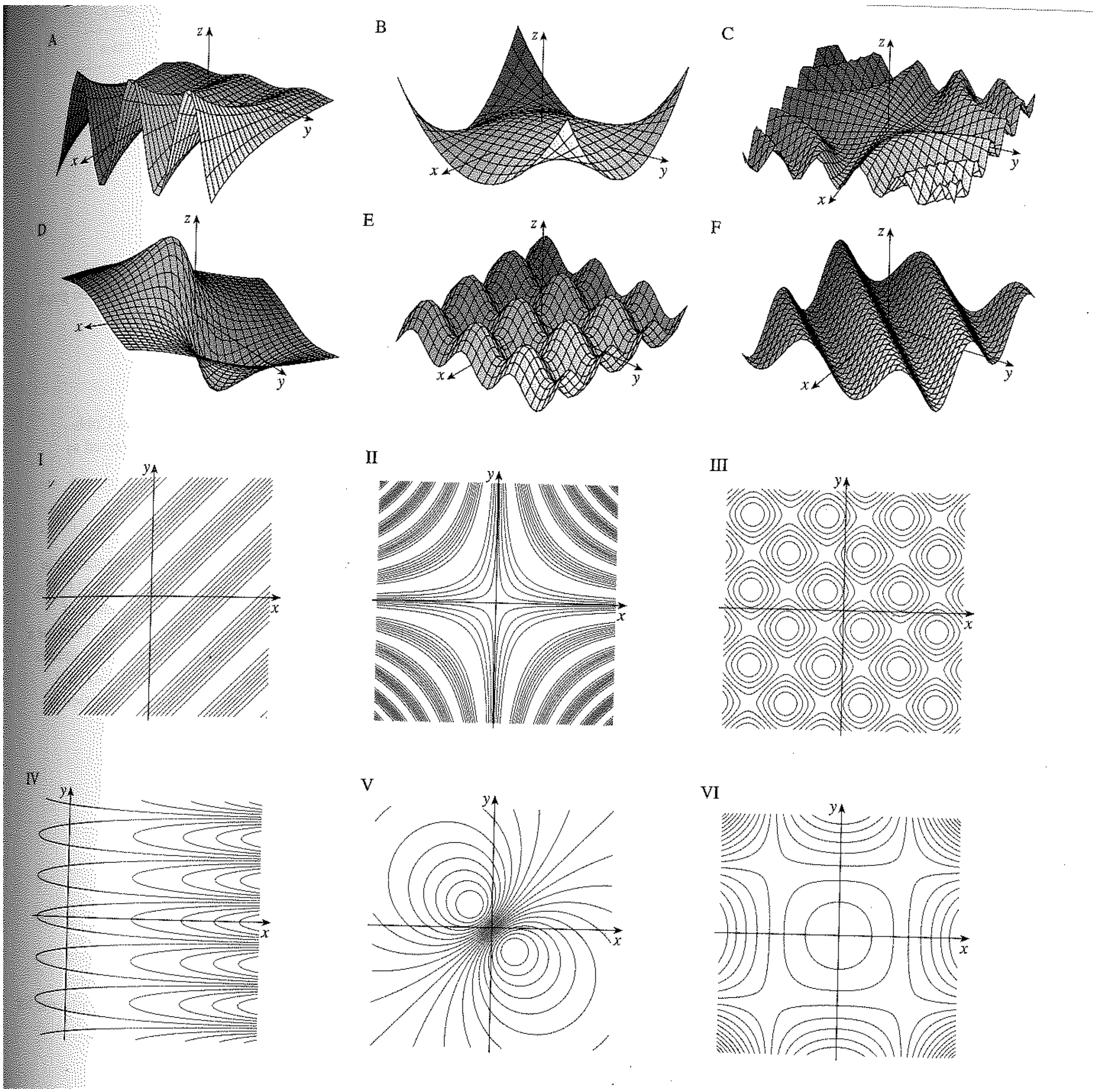
6 *14.1.19* Find and sketch the domain of the function $f(x,y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$

7 *14.1.69* Describe the level surfaces of the function $f(x,y,z) = y^2 + z^2$.

8 *14.1.71,72* Describe how the graph of g is obtained from the graph of f.

(a) $g(x,y) = f(x,y) + 2$
(b) $g(x,y) = -f(x,y)$
(c) $g(x,y) = f(x,y+2)$
(d) $g(x,y) = f(x+3,y-4)$

9 *14.1.61-66* Match the function with its graph (labeled A-F) and with its contour map (labeled I-VI). Give reasons for your choices.



MA 213 Worksheet #9

Section 14.3

1 *14.3.31* Find the first partial derivatives of $f(x, y, z) = x^3yz^2 + 2yz$.

2 *14.3.57* Find all second partial derivatives of $v = \sin(s^2 - t^2)$.

3 Verify that the conclusion of Clairaut's Theorem holds. That is, show $u_{xy} = u_{yx}$.
14.3.59 $u + x^4y^3 - y^4$
14.3.61 $u = \cos(x^2y)$.

4 *14.3.69* Find $\dfrac{\partial^3 w}{\partial z \partial y \partial x}$ and $\dfrac{\partial^3 w}{\partial x^2 \partial y}$ of $w = \dfrac{x}{y + 2z}$

5 *14.3.71* If $f(x, y, z) = xy^2z^3 + \arcsin(x\sqrt{z})$, find f_{xyz}. [*Hint:* Which order of differentiation is easiest?]

Additional Recommended Problems

6 *14.3.1* The temperature T (in $^\circ C$) at a location in the Northern Hemisphere depends on the longitude x, latitude y, and time t, so we can write $T = f(x, y, t)$.

(a) What are the meanings of the partial derivatives $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial t$?

(b) Honolulu has longitude 158° W and latitude 21° N. Suppose that at 9:00 a.m. on January 1 the wind is blowing hot air to the northeast, so the air to the west and south is warm, and the air to the north and east is cooler. Would you expect $f_x(158, 21, 9)$, $f_y(158, 21, 9)$, and $f_t(158, 21, 9)$ to be positive or negative? Explain.

7 *14.3.29* Find the first partial derivatives of $F(x, y) = \displaystyle\int_x^y \cos(e^t)\, dt$.

8 *14.3.77* Verify that the function $u = 1/\sqrt{x^2 + y^2 + z^2}$ is a solution of the three-dimensional laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.

MA 213 Worksheet #10

Section 14.4

1 *14.4.5* Find an equation of the tangent plane to the surface $z = x\sin(x+y)$ at the point $(-1,1,0)$.

2 *14.4.9* Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$ and $f_y(2,5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

3 *14.4.21* Find the linearization of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at (3,2,6) and use it to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.

4 *14.4.25* Find the differential of the function $z = e^{-2x}\cos 2\pi t$.

5 *14.4.33* The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

Additional Recommended Problems

6 *14.4.1* Find an equation of the tangent plane to the surface $z = 3y^2 - 2x^2$ at the point $(2,-1,-3)$.

7 *14.4.17* Verify the following linear approximation at $(0,0)$:

$$e^x \cos(xy) \approx x + 1.$$

8 *14.4.35* Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

9 *14.4.42* Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2,1,3)$. You don't have an equation for S but you know the curves

$$\mathbf{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t-t^2\rangle$$

$$\mathbf{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1\rangle$$

both lie on S. Find an equation of the tangent plane at P.

MA 213 Worksheet #11

Section 14.5

1 *14.5.1* Use the Chain Rule to find dz/dt for

$$z = xy^3 - x^2y, \quad x = t^2 + 1, \quad \text{and} \quad y = t^2 - 1$$

.

2 *14.5.11* Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$ for

$$z = e^r \cos(\theta), \quad r = st, \quad \text{and} \quad \theta = \sqrt{s^2 + t^2}.$$

3 *14.5.13* Let $p(t) = f(x, y)$, where f is differentiable, $x = g(t)$, $y = h(t)$, $g(2) = 4$, $g'(2) = -3$, $h(2) = 5$, $h'(2) = 6$, $f_x(4, 5) = 2$, $f_y(4, 5) = 8$. Find $p'(2)$.

4 *14.5.19* Use a tree diagram to write out the Chain Rule for the following. Assume all functions are differentiable.

$$\begin{array}{lll} T = F(p, q, r) & \text{where} & p = p(x, y, z) \text{ and} \\ r = r(x, y, z) & & q = q(x, y, z). \end{array}$$

5 *14.5.31* Find $\partial z/\partial x$ and $\partial z/\partial y$ assuming z is defined implicitly as a function of x and y as

$$x^2 + 2y^2 + 3z^2 = 1.$$

6 *14.5.39* Due to strange and difficult-to-explain circumstances, the length ℓ, width w, and height h of a box change with time. At a certain instant the dimensions are $\ell = 1$ m and $w = h = 2$ m, and ℓ and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s . At that instant find the rates at which the following quantities are changing.

(a) The volume
(b) The surface area
(c) The length of a diagonal

Additional Recommended Problems

7 *14.5.3* Use the Chain Rule to find dz/dt for $z = \sin(x)\cos(y)$, $x = \sqrt{t}$ and $y = 1/t$.

8 *14.5.15* Suppose f is a differentiable function of x and y, and $g(u, v) = f(e^u + \sin(v), e^u + \cos(v))$. Use the table of values to calculate $g_u(0, 0)$ and $g_v(0, 0)$.

	f	g	f_x	f_y
$(0, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

9 *14.5.23* Use the Chain Rule to find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ when $r = 2$, $\theta = \pi/2$.

$w = xy + yz + zx \quad x = r\cos(\theta) \quad y = r\sin(\theta) \quad z = r\theta$

10 *14.5.33* Find $\partial z/\partial x$ and $\partial z/\partial y$ assuming z is defines implicitly as a function of x and y as: $e^z = xyz$.

MA 213 Worksheet #12

Sections 14.6 and 14.7 (local extrema)

1 *14.6.10* $f(x, y, z) = y^2 e^{xyz}$, $\quad P(0, 1, -1)$, $\quad \mathbf{u} = \langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13} \rangle$

(a) Find the gradient of f.

(b) Evaluate the gradient at the point P.

(c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

2 (a) *14.6.13* Find the directional derivative of the function $g(s, t) = s\sqrt{t}$ at the point $(2, 4)$ in the direction of vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$.

(b) *14.6.20* Find the directional derivative of $f(x, y) = xy^2z^3$ at $P(2, 1, 1)$ in the direction of $Q(0, -3, 5)$.

3 *14.6.33* Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.

(a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\mathbf{v} = \mathbf{i}+\mathbf{j}-\mathbf{k}$.

(b) In which direction does V change most rapidly at P?

(c) What is the maximum rate of change at P?

4 Find the local maximum and minimum values and saddle point(s) of the function.
14.7.5 $f(x, y) = x^2 + xy + y^2 + y$
14.7.7 $f(x, y) = (x - y)(1 - xy)$
14.7.15 $f(x, y) = e^x \cos y$

5 *14.7.23* Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = x^2 + y^2 + x^{-2}y^{-2}$ precisely, using calculus.

Additional Recommended Problems

6 *14.6.15* Find the directional derivative of the function $f(x, y, z) = x^2y + y^2z$ at the point $(1, 2, 3)$, in the direction of vector $\mathbf{v} = \langle 2, -1, 2 \rangle$.

7 *14.6.42* Find equations of (a) the tangent plane and (b) the normal line to the given level surface at the point $(3, 1, -1)$.

8 *14.6.55* Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

9 *14.7.31* Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set D, where D is the closed triangle with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$.

10 *14.7.43* Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

MA 213 Worksheet #13

Section 14.7(global extrema) and 14.8

1 *14.7.33* Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the set $D = \{(x, y) \mid |x| \le 1, |y| \le 1\}$.

2 *14.7.53* A cardboard box without a lid is to have a volume of $32,000\ \text{cm}^3$. Find the dimensions that minimize the amount of cardboard used.

3 *14.7.42* Find the point on the plane $x - 2y + 3z = 6$ that is closest to the point $(0, 1, 1)$. Note: The distance to a plane formula on page 830 in the textbook can be used to check your answer.

4 Use Lagrange multipliers to find the absolute maximum and minimum values of the function subject to the given constraint.

(a) *14.8.3* $f(x, y) = x^2 - y^2, \quad x^2 + y^2 = 1$

(b) *14.8.9* $f(x, y, z) = xy^2z, \quad x^2 + y^2 + z^2 = 4$

5 *14.8.17* Find the extreme value of $f(x, y, z) = yz + xy$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$.

Additional Recommended Problems

6 *14.7.45* Find three positive numbers whose sum is 100 and whose product is a maximum.

7 *14.7.55* If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?

8 *14.8.23* Find the extreme values of $f(x, y) = e^{-xy}$ on the region described by the inequality $x^2 + 4y^2 \le 1$

9 *14.8.29* Use Lagrange multipliers to prove that the rectangle of maximum area that has a given perimeter p is a square.

MA 213 Worksheet #14

Section 15.1 and 15.2

1 Calculate the iterated integral.

(a) *15.1.15* $\int_1^4 \int_0^2 (6x^2y - 2x)\, dy\, dx$

(b) *15.1.17* $\int_0^1 \int_1^2 (x + e^{-y})\, dx\, dy$

2 *15.1.37* Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = \{(x, y) | \; -1 \le x \le 2, \, -1 \le y \le 1\}$

3 *15.2.13* Evaluate the double integral in two ways.

$$\iint_D x\, dA,$$

D is enclosed by the lines $y = x$, $y = 0$, $x = 1$.

4 *15.2.15* Set up iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why its easier.

$$\iint_D y\, dA$$

D is bounded by $y = x - 2$, $x = y^2$

5 *15.2.23* Find the volume of the solid that is under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Additional Recommended Problems

6 *15.1.9* Evaluate the double integral by first identifying it as the volume of a solid.

$$\iint_R \sqrt{2}\, dA, \quad R = \{(x, y) \mid 2 \le x \le 6, \; -1 \le y \le 5\}$$

7 *15.1.41* Find the volume of the solid enclosed by the surface $z = 1 + x^2ye^y$ and the planes $z = 0$, $x = \pm 1$, $y = 0$, and $y = 1$.

8 *15.2.1* Evaluate the iterated integral: $\int_1^5 \int_0^x (8x - 2y)\, dy\, dx$

9 *15.2.11* Draw an example of a region that is

(a) type I but not type II;

(b) type II but not type I.

10 *15.2.53* Sketch the region of integration, the evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1}\, dydx.$$

MA 213 Worksheet #15

Sections 15.3 and 15.6

1 *15.3.6* Sketch the region whose area is given by the following integral. Evaluate the integral.

$$\int_{\pi/2}^{\pi} \int_{0}^{2\sin\theta} r\, dr\, d\theta$$

2 *15.3.9* Evaluate the integral by changing to polar coordinates: $\iint_R \sin(x^2 + y^2)\, dA$, where R is the region in the first quadrant between the circles centered at the origin and radii 1 and 3.

3 Use a double integral to find the area of the given region.

15.3.15 One loop of the rose $r = \cos(3\theta)$.

15.3.17 inside the circle $(x-1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$.

4 *15.6.3* Evaluate the iterated integral: $\int_0^2 \int_0^{z^2} \int_0^{y-z} (2x - y)\, dx\, dy\, dz$

5 *15.6.9* Evaluate the triple integral:

$$\iiint_E y\, dV, \text{ where } E = \{(x, y, z) \mid 0 \le x \le 3,\ 0 \le y \le x,\ x - y \le z \le x + y\}.$$

6 *15.6.27* Sketch the solid whose volume is given by the integral $\int_0^1 \int_0^{1-x} \int_0^{2-2z} dy\, dz\, dx$

Additional Recommended Problems

7 *15.3.19* Use polar coordinates to find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \le 25$

8 *15.3.23* Use polar coordinates to find the volume of a sphere of radius a.

9 *15.3.29* Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx.$$

10 *15.6.35* Write the five other iterated integrals that are equal to the iterated integral,

$$\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy.$$

MA 213 Worksheet #16

Sections 15.6 and 15.7

1 *15.6.15* Evaluate the integral $\iiint_T y^2 V$, where T is the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$.

2 *15.6.21* Use a triple integral to find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

3 *15.7.1* Plot the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point.

(a) $(4, \pi/3, -2)$

(b) $(2, -\pi/2, 1)$

4 *15.7.3* Change from rectangular to cylindrical coordinates.

(a) $(-1, 1, 1)$

(b) $(-2, 2\sqrt{3}, 3)$

5 Use cylindrical coordinates to evaluate the following integrals.

15.7.17 $\iiint_E \sqrt{x^2 + y^2}\, dV$ where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

15.7.19 $\iiint_E (x + y + z)\, dV$, where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

Additional Recommended Problems

6 *15.6.13* Evaluate the triple integral:

$$\iiint_E 6xy\ dV,$$

where E is the (three dimensional) region that lies under the plane $z = 1 + x + y$ and above the (two dimensional) region in the xy-plane that is bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 1$.

7 *15.7.21* Evaluate $\iiint_E x^2\, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

8 *15.7.29* Evaluate the integral by changing to cylindrical coordinates.

$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{x^2+y^2}^{2} xz\ dz\ dx\ dy.$$

MA 213 Worksheet #17

Section 15.8

1 *15.8.1* Plot the point whose spherical coordinates are given. Then find the rectangular coordinates of the point.

(a) $(6, \pi/3, \pi/6)$

(b) $(3, \pi/2, 3\pi/4)$

2 *15.8.17* Sketch the solid whose volume is given by the integral and evaluate the integral.

$$\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

3 *15.8.25* Evaluate $\iiint_E xe^{x^2+y^2+z^2} dV$, where E is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant.

4 *15.8.29* Find the volume of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4\cos\phi$.

5 *15.8.41* Evaluate the integral by changing to spherical coordinates.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx$$

Additional Recommended Problems

6 *15.8.3* Change from rectangular to spherical coordinates.

(a) $(0, -2, 0)$

(b) $(-1, 1 - \sqrt{2})$

7 Identify the surface whose equation is given in spherical coordinates.

(a) *15.8.5* $\phi = \pi/3$

(b) *15.8.7* $\rho\cos\phi = 1$

8 *15.8.13* Sketch the solid described by the following inequalities.

$$2 \leq \rho \leq 4, \quad 0 \leq \phi \leq \pi/3, \quad 0 \leq \theta \leq \pi.$$

9 *15.8.35* Find the volume and centroid of the solid E that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

MA 213 Worksheet #18

Section 15.9

1 *15.9.1* Find the Jacobian of the transformation:

$$x = 2u + v, \quad y = 4u - v.$$

2 *15.9.9* Let S be the triangular region with vertices $(0,0), (1,1), (0,1)$. Find the image of S under the the transformation

$$x = u^2, \quad y = v.$$

3 *15.9.17* Evaluate the integral $\iint_R x^2\, dA$, where R is the region bounded by the ellipse $9x^2+4y^2 = 36$ using the transformation $x = 2u,\ y = 3v$.

4 *15.9.23* Evaluate the integral by making an appropriate change of variables: $\iint_R \frac{x-2y}{3x-y} dA$, where R is the parallelogram enclosed by the lines $x-2y = 0,\ x-2y = 4,\ 3x-y = 1,$ and $3x-y = 8$.

Additional Recommended Problems

5 Find the Jacobian of the transformations

(a) *15.9.3* $\quad x = s\cos t, \quad y = s\sin t$

(b) *15.9.5* $x = uv, \quad y = vw, \quad z = wu.$

6 *15.9.15* Evaluate the integral $\iint_R (x - 3y)\, dA$, where R is the triangular region with vertices $(0,0),\ (2,1),$ and $(1,2)$, using the transformation $x = 2u + v, \quad y = u + 2v$.

7 *15.9.21*

(a) Evaluate $\iiint_E dV$ where E is the solid enclosed by the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 = 1$. Use the transformation $x = au,\ y = bv,\ z = cw$.

(b) The earth is not a perfect sphere; rotation has resulted in flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of the earth.

8 *15.9.25* Evaluate the integral by making an appropriate change of variables: $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1,0),\ (2,0),\ (0,2),\ \text{and}(0,1)$.

MA 213 Worksheet #19

Section 16.1

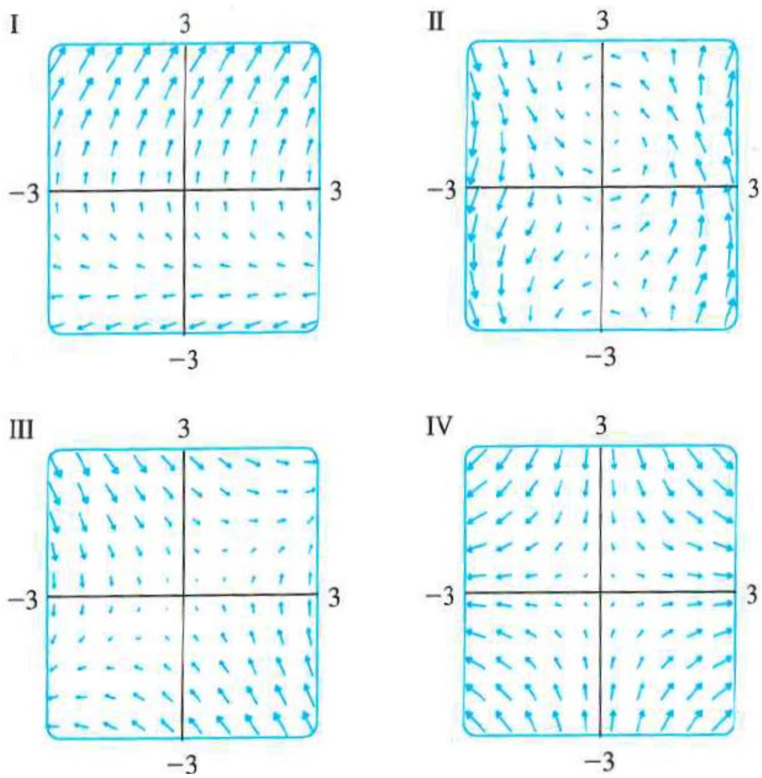
1 *16.1.11-14* Match the vector fields, $\mathbf{F}$, with the plots below. Give reasons for your choices.

(a) $\mathbf{F}(x,y) = \langle x, -y\rangle$

(b) $\mathbf{F}(x,y) = \langle y, x-y\rangle$

(c) $\mathbf{F}(x,y) = \langle y, y+2\rangle$

(d) $\mathbf{F}(x,y) = \langle \cos(x+y), x\rangle$



2 *16.1.23* Find the gradient vector field of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

3 *16.1.25* Let $f(x,y) = \frac{1}{2}(x-y)^2$. Find the gradient vector field, ∇f, of f and sketch it.

4 *16.1.33* A particle moves in a velocity field $\mathbf{V}(x,y) = \langle x^2, x+y^2\rangle$. If it is at position $(2,1)$ and time $t = 3$, estimate its location at time $t = 3.01$.

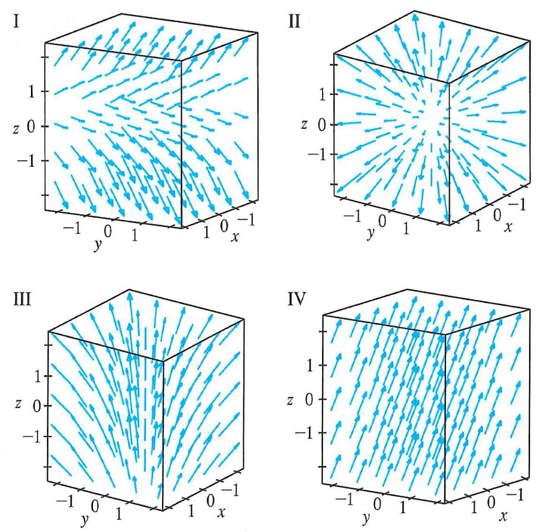
5 *16.1.15-18* Match the vector fields, $\mathbf{F}$, with the plots below. Give reasons for your choices.

(a) $\mathbf{F}(x,y,z) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

(b) $\mathbf{F}(x,y,z) = \mathbf{i} + 2\mathbf{j} + z\mathbf{k}$

(c) $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + 3\mathbf{k}$

(d) $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$



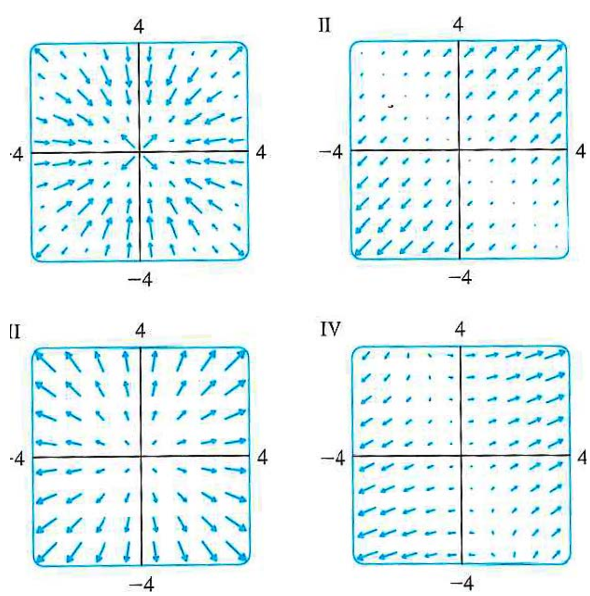
6 *16.1.29-32* Match the functions, f, with the plots of their gradient vector fields below. Give reasons for your choices.

(a) $f(x,y) = x^2 + y^2$

(b) $f(x,y) = x(x+y)$

(c) $f(x,y) = (x+y)^2$

(d) $f(x,y) = \sin\sqrt{x^2+y^2}$



MA 213 Worksheet #20

Section 16.2

1 Evaluate the line integral, where C is the given curve.

(a) *16.2.1* $\int_C y ds, \quad C : x = t^2,\ y = 2t,\ 0 \leq t \leq 3.$

(b) *16.2.5* $\int_C \left(x^2 y + \sin x\right)\ dy, \quad C$ is the arc of the parabola $y = x^2$ from $(0, 0)$ to (π, π^2).

2 Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the function $\mathbf{r}(t)$.

(a) *16.2.19* $\mathbf{F}(x, y) = xy^2\mathbf{i} - x^2\mathbf{j}, \quad \mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j},\ 0 \leq t \leq 1.$

(b) *16.2.22* $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + xy\mathbf{k}, \quad \mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k},\ 0 \leq t \leq \pi.$

3 *16.2.39* Find the work done by the force field $\mathbf{F}(x, y) = x\mathbf{i} + (y + 2)\mathbf{j}$ in moving an object along an arch of the cycloid: $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}, \quad 0 \leq t \leq 2\pi.$

4 *16.2.43* The position of an object with mass m at time t is $\mathbf{r}(t) = at^2\mathbf{i} + bt^3\mathbf{j},\ 0 \leq t \leq 1.$

(a) What is the force acting on the object at time t?

(b) What is the work done by the force during the time interval $0 \leq t \leq 1$?

Additional Recommended Problems

5 Evaluate the line integral, where C is the given curve.

(a) *16.2.8* $\int_C x^2\ dx + y^2\ dy$, C consists of the arc of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$ followed by the line segment from $(0, 2)$ to $(4, 3)$.

(b) *16.2.10* $\int_C y^2 z ds$, C is the line segment from $(3, 1, 2)$ to $(1, 2, 5)$.

(c) *16.2.14* $\int_C y dx + z dy + x dz$, $C : x = \sqrt{t}, y = t, z = t^2, 1 \leq t \leq 4.$

6 *16.2.33* A thin wire is bent in the shape of a semicircle $x^2 + y^2 = 4$, $x \geq 0$. If the linear density is a constant k, find the mass and center of mass of the wire.

7 *16.2.50* If C is a smooth curve given by a vector function $\mathbf{r}(t)$, $a \leq t \leq b$, show that

$$\int_C \mathbf{r} \cdot d\mathbf{r} = \frac{1}{2}\left[|\mathbf{r}(b)|^2 - |\mathbf{r}(a)|^2\right].$$

MA 213 Worksheet #21

Sections 16.3 and 16.4

1 *16.3.3* Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

16.3.3 $\mathbf{F}(x, y) = (xy + y^2)\mathbf{i} + (x^2 + 2xy)\mathbf{j}$.

16.3.7 $\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$

2 *16.3.12* Find a function f such that $\mathbf{F} = \nabla f$ and evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F}(x, y) = (3 + 2xy^2)\mathbf{i} + 2x^2y\mathbf{j},$$

and C is the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $(4, \frac{1}{4})$.

3 *16.3.19* Show the line integral $\displaystyle\int_C 2xe^{-y}dx + (2y - x^2e^{-y})$, where C is any path from $(1, 0)$ to (2,1), is independent of path and evaluate the integral.

4 *16.4.1* Evaluate the line integral $\displaystyle\oint_C y^2\,dx + x^2y\,dy$ where C is the rectangle with vertices $(0, 0), (5, 0), (5, 4)$, and $(0, 4)$ by two methods:

(i) directly and

(ii) using Green's Theorem.

5 *16.4.7* Use Green's Theorem to evaluate $\displaystyle\oint_C (y+e^{\sqrt{x}})\,dx+(2x+\cos y^2)\,dy$ where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

6 *16.4.13* Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y - \cos y, x\sin y\rangle$ and C is the circle $(x - 3)^2 + (y + 4)^2 = 4$ oriented clockwise.

Additional Recommended Problems

7 *16.3.15* Find a function f such that $\mathbf{F} = \nabla f$ and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k},$$

and C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$.

8 *16.3.23* Find the work done by the force field $\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}$ in moving an object from $P(1, 0)$ to $Q(2, 2)$.

9 *16.4.11*Use Green's Theorem to evaluate $\displaystyle\oint_C \langle y\cos c - xy, xy + x\cos x\rangle \cdot\ d\mathbf{r}$, where C is the triangle from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$. Be sure to check the orientation of the curve before applying the theorem.

10 *16.4.17* Use Green's Theorem to find the work done by the force $\mathbf{F}(x, y) = x(x + y)\mathbf{i} + xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to $(1, 0)$, then along the line segment to $(0, 1)$ and then back to the origin along the y-axis.

MA 213 Worksheet #22

Section 16.5

1 Find (1) the curl and (2) the divergence of the vector field.

16.5.1 $\mathbf{F}(x, y, z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$.

16.5.7 $\mathbf{F}(x, y, z) = \langle e^x \sin y, e^y \sin z, e^z \sin x\rangle$

2 *16.5.12* Let f be a scalar field and $\mathbf{F}$ a vector field. State whether each expression is meaningful. If no, explain why. If so, state whether is is a scalar field or vector field.

(a) curl(curl $\mathbf{F}$)

(b) div(div $\mathbf{F}$)

(c) curl(grad f)

(d) (grad f) $\times$ (div $\mathbf{F}$)

(e) grad(div f)

(f) div(curl(grad f))

3 *16.5.15* Use curl $\mathbf{F}$ to determine whether or not the vector field $\mathbf{F}(x, y, z) = z\cos(y)\mathbf{i}+xz\sin(y)\mathbf{j}+x\cos(y)\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

4 *16.5.23* Let $\mathbf{F}$ and $\mathbf{G}$ be vector fields. Prove the identity, assuming that the appropriate partial derivatives exist and are continuous.

$$\text{div } (\mathbf{F} + \mathbf{G}) = \text{div } \mathbf{F} + \text{div } \mathbf{G}.$$

Additional Recommended Problems

5 *16.6.17* Use curl $\mathbf{F}$ to determine whether or not the vector field $\mathbf{F}(x, y, z) = e^{yz}\mathbf{i}+xze^{yz}\mathbf{j}+xye^{yz}\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

6 *16.5.25* Prove the identity, assuming that the appropriate partial derivatives exists and are continuous.

$$\text{div}(f\mathbf{F}) = f\text{div}\mathbf{F} + \mathbf{F}\cdot\nabla f$$

7 *16.5.31* Let $\mathbf{r} = \langle x, y, z\rangle$ and $r = |\mathbf{r}|$. Verify each identity.

(a) $\nabla\mathbf{r} = \mathbf{r}/r$

(b) $\nabla\times\mathbf{r} = \mathbf{0}$

(c) $\nabla(1/r) = -\mathbf{r}/r^3$

(d) $\nabla\ln r = \mathbf{r}/r^2$

MA 213 Worksheet #23

Section 16.6

1 *16.6.5* Identify the surface with the vector equation:

$$\mathbf{r}(s,t) = \langle s\cos t, s\sin t, s\rangle.$$

2 *16.6.21* Find a parametric representation for the part of the hyperboloid $4x^2 - 4y^2 - z^2 = 4$ that lies in front of the yz-plane.

3 *16.6.37* Find an equation of the tangent plane to the parametric surface

$$\mathbf{r}(u,v) = \langle u^2, 2u\sin v, u\cos v\rangle,$$

at the point $u = 1, v = 0$.

4 Find the surface area.

16.6.47 The part of the paraboloid $y = x^2 + z^2$ that lies within the cylinder $x^2 + z^2 = 16$.

16.6.49 The surface with parametric equations $x = u^2,\ \ y = uv,\ \ z = \frac{1}{2}v^2;\ \ 0 \le u \le 1,\ \ 0 \le v \le 2.$

Additional Recommended Problems

5 *16.6.3* Identify the surface with the given vector equation:

$$\mathbf{r}(u,v) = (u+v)\mathbf{i} + (3-v)\mathbf{j} + (1+4u+5v)\mathbf{k}.$$

6 *16.6.23* Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

7 *16.6.33* Find an equation of the tangent plane to the given parametric surface at the specified point.

$$x = u + v,\ \ y = 3u^2,\ \ z = u - v;\ \ (2,3,0).$$

8 *16.6.59*

(a) Show that the parametric equations $x = a\sin u\cos v,\ y = b\sin u\sin v,\ z = c\cos u$ for $0 \le u \le \pi$ and $0 \le v \le 2\pi$, represent a ellipsoid.

(b) Set up, but do not evaluate, a double integral for the surface area of the ellipsoid in part (a).

MA 213 Worksheet #24

Section 16.7

1 *16.7.5* Evaluate the surface integral $\iint_S (x+y+z)\ dS$ where S is the parallelogram with parametric equations $x = u+v,\ y = u-v,\ z = 1+2u+v$ where $0 \le u \le 2$ and $0 \le v \le 1$.

2 *16.7.19* Evaluate the surface integral

$$\iint_S xz\ dS,$$

where S is the boundary of the region enclosed by the cylinder $y^2+z^2 = 9$ and the planes $x = 0$ and $x + y = 5$.

3 *16.7.31* Evaluate the surface integral $\iint_S \mathbf{F} \cdot\ d\mathbf{S}$ where $\mathbf{F}$ is the vector field

$$\mathbf{F}(x,y,z) = \langle x^2, y^2, z^2 \rangle$$

and the oriented surface S is the boundary of the solid half-cylinder $0 \le z \le \sqrt{1-y^2},\ 0 \le x \le 2$. (In other words, find the flux of $\mathbf{F}$ across S.)

Additional Recommended Problems

4 *16.6.11* Evaluate the surface integral

$$\iint_S x\ dS,$$

where S is the triangular region with vertices $(1,0,0), (0,-2,0)$, and $(0,0,4)$.

5 *16.7.21* Evaluate the surface integral $\iint_S \mathbf{F} \cdot\ d\mathbf{S}$ where $\mathbf{F}$ is the vector field $\mathbf{F} = ze^{xy}\mathbf{i} - 3ze^{xy}\mathbf{j} + xy\mathbf{k}$ and the oriented surface S is the parallelogram of problem 1, with upward orientation. (In other words, find the flux of $\mathbf{F}$ across S.)

6 *16.7.45* Use Gauss's Law to find the charge contained in the solid hemisphere $x^2+y^2+z^2 \le a^2,\ z \ge 0$, if the electric field is

$$\mathbf{E}(x,y,z) = \langle x, y, 2z \rangle.$$

MA 213 Worksheet #25

Section 16.8

1 *16.8.3* Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ for

$$\mathbf{F}(x, y, z) = ze^y\mathbf{i} + x\cos(y)\mathbf{j} + xz\sin(y)\mathbf{k},$$

where S is the hemisphere $x^2 + y^2 + z^2 = 16, y \geq 0$, oriented in the direction of the positive y-axis.

2 *16.8.7* Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ (where C is oriented counterclockwise as viewed from above) for

$$\mathbf{F}(x, y, z) = (x + y^2)\mathbf{i} + (y + z^2)\mathbf{j} + (z + x^2)\mathbf{k},$$

where C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$

3 *16.8.13* Verify that Stokes' Theorem is true for the vector field $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} - 2\mathbf{k}$ and surface S is the cone $z^2 = x^2 + y^2$, $0 \leq z \leq 4$, oriented downward.

Additional Recommended Problems

4 *16.8.5* Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, for $\mathbf{F}(x, y, z) = xyz\mathbf{i} + xy\mathbf{j} + x^2yz\mathbf{k}$, S consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, oriented outward

5 *16.8.10* Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ (where C is oriented counterclockwise as viewed from above) for $\mathbf{F}(x, y, z) = 2y\mathbf{i} + xz\mathbf{j} + (x + y)\mathbf{k}$, and C is the curve of intersection of the plane $z = y + 2$ and the cylinder $x^2 + y^2 = 1$.

6 *16.8.19* If S is a sphere and $\mathbf{F}$ satisfies the hypotheses of Stokes' Theorem, show that $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$

MA 213 Worksheet #26

Section 16.9

1 *16.9.3* Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = \langle z, y, x \rangle$ and E the solid ball $x^2 + y^2 + z^2 \leq 16$.

2 *16.9.5* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$\mathbf{F}(x, y, z) = xye^z\mathbf{i} + xy^2z^3\mathbf{j} - ye^z\mathbf{k},$$

and S is the surface of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$.

3 *16.9.7* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$$

and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

Additional Recommended Problems

4 *16.9.1* Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$ and E the cube bounded by the planes $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0$, and $z = 1$.

5 *16.9.11* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$\mathbf{F}(x, y, z) = (2x^3 + y^3)\mathbf{i} + (y^3 + z^3)\mathbf{j} + 3y^2z\mathbf{k}$$

where S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.

6 *16.9.25* Prove the following identity, assuming that S and E satisfy the conditions of the Divergence Theorem and the scalar functions and components of the vector fields have continuous second-order partial derivatives:

$$\iint_S \mathbf{a} \cdot \mathbf{n}\, dS = 0 \text{ where } \mathbf{a} \text{ is a constant vector.}$$