MA 213 Worksheet #4
Section 12.4

1 Find the cross product $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. Verify that $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
   12.4.2 $\mathbf{a} = \langle 4, 3, -2 \rangle,$ $\mathbf{b} = \langle 2, -1, 1 \rangle$
   12.4.5 $\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k},$ $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

2 *12.4.20* Find two unit vectors orthogonal to both $\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$.

3 *12.4.29* For points $P(1, 0, 1)$, $Q(-2, 1, 3)$, and $R(4, 2, 5)$

   (a) Find a nonzero vector orthogonal to the plane through the points P, Q, and R;

   (b) Find the area of triangle PQR.

4 *12.4.34* Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Are these vectors coplanar?

5 *12.4.41* A wrench 30cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction $\langle 0, 3, -4 \rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100N·m of troque to the bolt.

6 *12.4.43* If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Additional Recommended Problems

7 *12.4.17* If $\mathbf{a} = \langle 2, -1, 3 \rangle$ and $\mathbf{b} = \langle 4, 2, 1 \rangle$, find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

8 *12.4.22* Explain why $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$ for all vectors $\mathbf{a}$ and $\mathbf{b}$ in V_3.

9 *12.4.37* Use the scalar triple product to verify that the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar.

10 *12.4.44* (a) Find all vectors $\mathbf{v}$ such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, -5 \rangle$$

   (b) Explain why there is no vector $\mathbf{v}$ such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, 5 \rangle$$