# MA 213 Worksheet #12

Sections 14.6 and 14.7 (local extrema)

**1** *14.6.10* $f(x,y,z) = y^2e^{xyz}$, $\quad P(0,1,-1)$, $\quad \mathbf{u} = \langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13} \rangle$

(a) Find the gradient of $f$.

(b) Evaluate the gradient at the point $P$.

(c) Find the rate of change of $f$ at $P$ in the direction of the vector $\mathbf{u}$.

**2** (a) *14.6.13* Find the directional derivative of the function $g(s,t) = s\sqrt{t}$ at the point $(2,4)$ in the direction of vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$.

(b) *14.6.20* Find the directional derivative of $f(x,y) = xy^2z^3$ at $P(2,1,1)$ in the direction of $Q(0,-3,5)$.

**3** *14.6.33* Suppose that over a certain region of space the electrical potential $V$ is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.

(a) Find the rate of change of the potential at $P(3,4,5)$ in the direction of the vector $\mathbf{v} = \mathbf{i}+\mathbf{j}-\mathbf{k}$.

(b) In which direction does $V$ change most rapidly at $P$?

(c) What is the maximum rate of change at $P$?

**4** Find the local maximum and minimum values and saddle point(s) of the function.
*14.7.5* $f(x,y) = x^2 + xy + y^2 + y$
*14.7.7* $f(x,y) = (x-y)(1-xy)$
*14.7.15* $f(x,y) = e^x \cos y$

**5** *14.7.23* Find the local maximum and minimum values and saddle point(s) of the function $f(x,y) = x^2 + y^2 + x^{-2}y^{-2}$ precisely, using calculus.

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**Additional Recommended Problems**

**6** *14.6.15* Find the directional derivative of the function $f(x,y,z) = x^2y + y^2z$ at the point $(1,2,3)$, in the direction of vector $\mathbf{v} = \langle 2,-1,2 \rangle$.

**7** *14.6.42* Find equations of (a) the tangent plane and (b) the normal line to the given level surface at the point $(3,1,-1)$.

**8** *14.6.55* Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

**9** *14.7.31* Find the absolute maximum and minimum values of $f(x,y) = x^2 + y^2 - 2x$ on the set $D$, where $D$ is the closed triangle with vertices $(2,0)$, $(0,2)$, and $(0,-2)$.

**10** *14.7.43* Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4,2,0)$.