MA 213 Worksheet #18

Section 15.9

1 *15.9.1* Find the Jacobian of the transformation:

$$x = 2u + v, \quad y = 4u - v.$$

2 *15.9.9* Let S be the triangular region with vertices $(0,0), (1,1), (0,1)$. Find the image of S under the the transformation

$$x = u^2, \quad y = v.$$

3 *15.9.17* Evaluate the integral $\iint_R x^2\, dA$, where R is the region bounded by the ellipse $9x^2+4y^2 = 36$ using the transformation $x = 2u,\ y = 3v$.

4 *15.9.23* Evaluate the integral by making an appropriate change of variables: $\iint_R \frac{x-2y}{3x-y} dA$, where R is the parallelogram enclosed by the lines $x-2y = 0,\ x-2y = 4,\ 3x-y = 1,$ and $3x-y = 8$.

Additional Recommended Problems

5 Find the Jacobian of the transformations

(a) *15.9.3* $\quad x = s\cos t, \quad y = s\sin t$

(b) *15.9.5* $x = uv, \quad y = vw, \quad z = wu.$

6 *15.9.15* Evaluate the integral $\iint_R (x - 3y)\, dA$, where R is the triangular region with vertices $(0,0),\ (2,1)$, and $(1,2)$, using the transformation $x = 2u + v, \quad y = u + 2v$.

7 *15.9.21*

(a) Evaluate $\iiint_E dV$ where E is the solid enclosed by the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 = 1$. Use the transformation $x = au,\ y = bv,\ z = cw$.

(b) The earth is not a perfect sphere; rotation has resulted in flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of the earth.

8 *15.9.25* Evaluate the integral by making an appropriate change of variables: $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1,0),\ (2,0),\ (0,2),$ and$(0,1)$.