MA 213 Worksheet #20

Section 16.2

1 Evaluate the line integral, where C is the given curve.

(a) *16.2.1* $\displaystyle\int_C y ds, \quad C: x = t^2,\ y = 2t,\ 0 \le t \le 3.$

(b) *16.2.5* $\displaystyle\int_C \left(x^2y + \sin x\right)\ dy, \quad C$ is the arc of the parabola $y = x^2$ from $(0,0)$ to (π, π^2).

2 Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the function $\mathbf{r}(t)$.

(a) *16.2.19* $\mathbf{F}(x,y) = xy^2\mathbf{i} - x^2\mathbf{j}, \quad \mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j},\ 0 \le t \le 1.$

(b) *16.2.22* $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + xy\mathbf{k}, \quad \mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k},\ 0 \le t \le \pi.$

3 *16.2.39* Find the work done by the force field $\mathbf{F}(x,y) = x\mathbf{i} + (y+2)\mathbf{j}$ in moving an object along an arch of the cycloid: $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}, \quad 0 \le t \le 2\pi.$

4 *16.2.43* The position of an object with mass m at time t is $\mathbf{r}(t) = at^2\mathbf{i} + bt^3\mathbf{j},\ 0 \le t \le 1.$

(a) What is the force acting on the object at time t?

(b) What is the work done by the force during the time interval $0 \le t \le 1$?

Additional Recommended Problems

5 Evaluate the line integral, where C is the given curve.

(a) *16.2.8* $\displaystyle\int_C x^2\ dx + y^2\ dy$, C consists of the arc of the circle $x^2 + y^2 = 4$ from $(2,0)$ to $(0,2)$ followed by the line segment from $(0,2)$ to $(4,3)$.

(b) *16.2.10* $\displaystyle\int_C y^2 z ds$, C is the line segment from $(3,1,2)$ to $(1,2,5)$.

(c) *16.2.14* $\displaystyle\int_C y dx + z dy + x dz$, $C: x = \sqrt{t}, y = t, z = t^2, 1 \le t \le 4.$

6 *16.2.33* A thin wire is bent in the shape of a semicircle $x^2 + y^2 = 4$, $x \ge 0$. If the linear density is a constant k, find the mass and center of mass of the wire.

7 *16.2.50* If C is a smooth curve given by a vector function $\mathbf{r}(t)$, $a \le t \le b$, show that

$$\int_C \mathbf{r} \cdot d\mathbf{r} = \frac{1}{2}\left[|\mathbf{r}(b)|^2 - |\mathbf{r}(a)|^2\right].$$