MA 213 Worksheet #21

Sections 16.3 and 16.4

1 *16.3.3* Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

16.3.3 $\mathbf{F}(x, y) = (xy + y^2)\mathbf{i} + (x^2 + 2xy)\mathbf{j}$.

16.3.7 $\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$

2 *16.3.12* Find a function f such that $\mathbf{F} = \nabla f$ and evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F}(x, y) = (3 + 2xy^2)\mathbf{i} + 2x^2y\mathbf{j},$$

and C is the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $(4, \frac{1}{4})$.

3 *16.3.19* Show the line integral $\displaystyle\int_C 2xe^{-y}dx + (2y - x^2e^{-y})$, where C is any path from $(1, 0)$ to (2,1), is independent of path and evaluate the integral.

4 *16.4.1* Evaluate the line integral $\displaystyle\oint_C y^2\,dx + x^2y\,dy$ where C is the rectangle with vertices $(0, 0), (5, 0), (5, 4)$, and $(0, 4)$ by two methods:

(i) directly and

(ii) using Green's Theorem.

5 *16.4.7* Use Green's Theorem to evaluate $\displaystyle\oint_C (y+e^{\sqrt{x}})\,dx+(2x+\cos y^2)\,dy$ where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

6 *16.4.13* Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y - \cos y, x\sin y\rangle$ and C is the circle $(x - 3)^2 + (y + 4)^2 = 4$ oriented clockwise.

Additional Recommended Problems

7 *16.3.15* Find a function f such that $\mathbf{F} = \nabla f$ and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k},$$

and C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$.

8 *16.3.23* Find the work done by the force field $\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}$ in moving an object from $P(1, 0)$ to $Q(2, 2)$.

9 *16.4.11*Use Green's Theorem to evaluate $\displaystyle\oint_C \langle y\cos c - xy, xy + x\cos x\rangle \cdot\ d\mathbf{r}$, where C is the triangle from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$. Be sure to check the orientation of the curve before applying the theorem.

10 *16.4.17* Use Green's Theorem to find the work done by the force $\mathbf{F}(x, y) = x(x + y)\mathbf{i} + xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to $(1, 0)$, then along the line segment to $(0, 1)$ and then back to the origin along the y-axis.