MA 213 Worksheet #22

Section 16.5

1 Find (1) the curl and (2) the divergence of the vector field.

16.5.1 $\mathbf{F}(x, y, z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$.

16.5.7 $\mathbf{F}(x, y, z) = \langle e^x \sin y, e^y \sin z, e^z \sin x\rangle$

2 *16.5.12* Let f be a scalar field and $\mathbf{F}$ a vector field. State whether each expression is meaningful. If no, explain why. If so, state whether is is a scalar field or vector field.

(a) curl(curl $\mathbf{F}$)

(b) div(div $\mathbf{F}$)

(c) curl(grad f)

(d) (grad f) $\times$ (div $\mathbf{F}$)

(e) grad(div f)

(f) div(curl(grad f))

3 *16.5.15* Use curl $\mathbf{F}$ to determine whether or not the vector field $\mathbf{F}(x, y, z) = z\cos(y)\mathbf{i}+xz\sin(y)\mathbf{j}+x\cos(y)\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

4 *16.5.23* Let $\mathbf{F}$ and $\mathbf{G}$ be vector fields. Prove the identity, assuming that the appropriate partial derivatives exist and are continuous.

$$\text{div } (\mathbf{F} + \mathbf{G}) = \text{div } \mathbf{F} + \text{div } \mathbf{G}.$$

Additional Recommended Problems

5 *16.6.17* Use curl $\mathbf{F}$ to determine whether or not the vector field $\mathbf{F}(x, y, z) = e^{yz}\mathbf{i}+xze^{yz}\mathbf{j}+xye^{yz}\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

6 *16.5.25* Prove the identity, assuming that the appropriate partial derivatives exists and are continuous.

$$\text{div}(f\mathbf{F}) = f\text{div}\mathbf{F} + \mathbf{F} \cdot \nabla f$$

7 *16.5.31* Let $\mathbf{r} = \langle x, y, z\rangle$ and $r = |\mathbf{r}|$. Verify each identity.

(a) $\nabla \mathbf{r} = \mathbf{r}/r$

(b) $\nabla \times \mathbf{r} = \mathbf{0}$

(c) $\nabla(1/r) = -\mathbf{r}/r^3$

(d) $\nabla \ln r = \mathbf{r}/r^2$