MA 213 Worksheet #26

Section 16.9

1 *16.9.3* Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = \langle z, y, x \rangle$ and E the solid ball $x^2 + y^2 + z^2 \leq 16$.

2 *16.9.5* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$\mathbf{F}(x, y, z) = xye^z\mathbf{i} + xy^2z^3\mathbf{j} - ye^z\mathbf{k},$$

and S is the surface of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$.

3 *16.9.7* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$$

and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

Additional Recommended Problems

4 *16.9.1* Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$ and E the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

5 *16.9.11* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for

$$\mathbf{F}(x, y, z) = (2x^3 + y^3)\mathbf{i} + (y^3 + z^3)\mathbf{j} + 3y^2z\mathbf{k}$$

where S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.

6 *16.9.25* Prove the following identity, assuming that S and E satisfy the conditions of the Divergence Theorem and the scalar functions and components of the vector fields have continuous second-order partial derivatives:

$$\iint_S \mathbf{a} \cdot \mathbf{n}\, dS = 0 \text{ where } \mathbf{a} \text{ is a constant vector.}$$