MA 213 Worksheet #27

Final Review

Unit IV Problems

1 Various line integrals

(a) *16.review.3(a)* Write the definition of the line integral of a scalar function f along a smooth curve C with respect to arc length.

(b) *16.review.3(d)* Write the definitions of the line integrals along C of a scalar function f with respect to x, y, and z.

(c) *16.review.4(a)* Define the line integral of a vector field $\mathbf{F}$ along a smooth curve C given by a vector function $\mathbf{r}(t)$.

2 Match up the following.

gradient	turns a vector to a vector
curl	turns a scalar to a vector
divergence	turns a vector to a scalar

3 Parametrizing

(a) How many parameters do you need to parameterize a curve? How many parameters do you need to parameterize a surface?

(b) How do you parameterize the line segment between two points $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$?

(c) How do you parameterize a circle $z^2 + x^2 = c$? What about $y^2 - z^2 = c$?

(d) How do you parameterize an ellipsoid $4x^2 + 9y^2 + 6z^2 = 36$?

4 *16.review.31* Verify that Stokes' Theorem is true for the vector field $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$, where S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane and S has upward orientation.

5 *16.review.35* Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, where E is the unit ball $x^2 + y^2 + z^2 \leq 1$

6 *16.review.13* Show that $\mathbf{F}$ is conservative and use this fact to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve. Here:

$$\mathbf{F}(x, y, z) = (4x^3y^2 - 2xy^3)\mathbf{i} + (2x^4y - 3x^2y^2 + 4y^3)\mathbf{j}$$

$$C\colon \mathbf{r}(t) = (t + \sin \pi t)\mathbf{i} + (2t + \cos \pi t)\mathbf{j},\ 0 \leq t \leq 1$$

Unit I-III Problems

7 (a) *15.review.7* How do you evaluate $\iiint_E f(x,y,z)dV$? How do you find the bounds using the bounded solid region E?

(b) *15.review.10* If a transformation T is given by $x = g(u,v)$, $y = h(u,v)$, what is the Jacobian of T? How do you change variables in a double integral? A triple integral?

(c) *14.review.13(a)* Write an expression as a limit for the directional derivative of f at (x_0, y_0) in the direction of a unit vector $\mathbf{u} = \langle a, b\rangle$.

(d) Given a surface $\mathbf{r}(u,v)$, how to find its tangent plane at a given point P?

8 *15.review.25* Calculate the value of the multiple integral.
$\iint_D ydA$, where D is the region in the first quadrant bounded by the parabolas $x = y^2$ and $x = 8 - y^2$.

9 *15.review.30* Calculate the value of the multiple integral.
$\iiint_T ydV$, where T is the solid tetrahedron with vertices $(0,0,0), (\frac{1}{3},0,0), (0,1,0)$ and $(0,0,1)$.

10 *15.review.47* Use polar coordinates to evaluate

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2)dydx$$

11 *14.review.35* If $u = x^2y^3 + z^4$, where $x = p + 3p^2$, $y = pe^p$, and $z = p\sin(p)$, use the Chain Rule to find du/dp.

12 *14.review.56* Find the absolute maximum and minimum values of f on the set D. $f(x,y) = e^{-x^2-y^2}(x^2 + 2y^2)$; D is the disk $x^2 + y^2 \leq 4$.

13 *14* Using Lagrange multipliers find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to the point $(3,1,-1)$.

14 *13.review.2* Let $r(t) = \langle \sqrt{2-t}, (e^t - 1)/t, \ln(t+1)\rangle$.

(a) Find the domain of r.

(b) Find $\lim_{t\to 0} r(t)$.

(c) Find $r'(t)$.

15 *13.4.7* Find the position vector of a particle that has acceleration vector $a(t) = 2t\mathbf{i} + \sin(t)\mathbf{j} + \cos(2t)\mathbf{k}$, initial velocity $v(0) = \mathbf{i}$, and initial position $r(0) = \mathbf{j}$.

16 *12.review.7* Assume $u \cdot (v \times w) = 2$. Find

(a) $(u \times v) \cdot w$

(b) $u \cdot (w \times v)$

17 *12* Find a vector perpendicular to the plane through the points $A = (1,0,0)$, $B = (2,0,-1)$, $C = (1,4,3)$. Find the area of the triangle ABC.