Math 213 Exam II Review

October 14, 2018

Functions of two variables: graphs, contour plots, limits, continuity

1. Find $\lim_{(x,y)\to(0,0)} \dfrac{x^4 + x^2y^2}{x^3y + y^4}$ or show that it does not exist.

2. Find the domain of the function $f(x, y) = \dfrac{\ln(16 - x^2 - y^2)}{x^2 + y^2 - 1}$.

3. At what points is the function

$$f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ \\ 1, & (x,y) = (0,0) \end{cases}$$

   continuous?

Partial Derivatives, Chain Rule

1. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ if $e^{xyz} = yz$

2. Show that the function $u(x, t) = t^{-1/2}e^{-x^2/t}$ solves the *heat equation*

$$u_t(x, t) = u_{xx}(x, t)$$

   for $x \in \mathbb{R}$ and $t > 0$.

3. Find the critical points of the function $f(x, y) = x^4 - 4x^3 + y^2 - 2y$.

Tangent Planes and Linear Approximation

1. Find the equation of the tangent plane to the graph of

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 3$$

at the point $(2, 3, 2)$.

2. Find the linear approximation to $f(x, y) = xe^y$ at $(1, 0)$. Use it to estimate $f(0.9, -0.1)$.

Gradient, Directional Derivatives

1. Find the maximum rate of change of $f(x, y) = x^2 - y^2$ at the point $(3, 2)$ and find the direction of the maximum rate of change.

2. Find the equation of the normal line to level set of

$$f(x, y, z) = \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4}$$

at the point $(2, 3, 2)$.

Maximum and Minimum Values

1. Find the critical points of $f(x, y) = x^3 - 6xy + 8y^3$ and classify them as maxima, minima, or saddle points.

2. Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region D in the xy-plane with vertices $(0, 0)$, $(0, 6)$, and $(6, 0)$.

3. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

Lagrange Multiplier Method

1. Find the maximum and minimum values of $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$

2. Find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 3$.