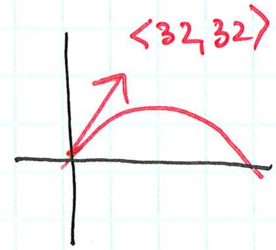
9/17/18 ①

1) $\vec{r}(t) = \langle 32t,\ 32t - 16t^2 \rangle$



$\vec{v}(t) = \langle \underline{32},\ \underline{32} - \underline{32t} \rangle$

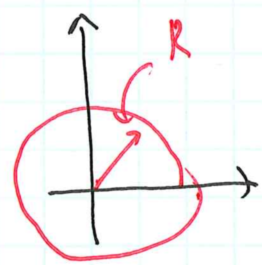
$\vec{a}(t) = \langle 0, -32 \rangle$

Note: $\vec{v}(0) = \langle 32, 32 \rangle$

2) $\vec{r}(t) = \left\langle R\cos\left(\frac{2\pi t}{T}\right),\ R\sin\left(\frac{2\pi t}{T}\right) \right\rangle$

$R =$ Radius

$T =$ Period



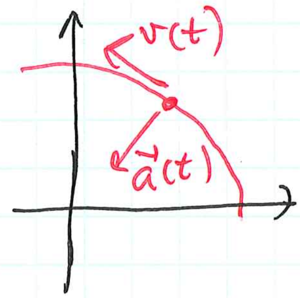
$$\vec{v}(t) = \left\langle R\left(-\sin\left(\frac{2\pi t}{T}\right)\right)\cdot \frac{2\pi}{T},\ R\cdot\cos\left(\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T} \right\rangle$$

$$= \left\langle \frac{2\pi R}{T}\left(-\sin\left(\frac{2\pi t}{T}\right)\right),\ \frac{2\pi R}{T}\cos\left(\frac{2\pi t}{T}\right) \right\rangle$$

$$= \frac{2\pi R}{T}\left\langle -\sin\left(\frac{2\pi t}{T}\right),\ \cos\left(\frac{2\pi t}{T}\right) \right\rangle$$

$$\vec{a}(t) = \left(\frac{2\pi R}{T}\right) \left\langle -\frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right), -\frac{2\pi}{T}\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$= \left(\frac{2\pi}{T}\right)^2 \cdot R \left\langle -\cos\left(\frac{2\pi t}{T}\right), -\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$



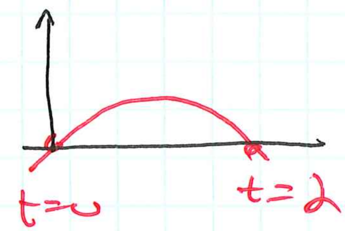
$$v = \frac{2\pi R}{T}$$

$$= \left(\frac{2\pi R}{T}\right)^2 \cdot \frac{1}{R} \left\langle \cos\left(\frac{2\pi t}{T}\right), -\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$= \frac{v^2}{R} \left\langle -\cos\left(\frac{2\pi t}{T}\right), -\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

1) Projectile

$$\vec{r}(t) = \langle 32t, 32t - 16t^2 \rangle$$



$$\vec{v}(t) = \langle 32, 32 - 32t \rangle$$

$$\vec{a}(t) = \langle 0, -32 \rangle$$

Time hits the ground:

$$32t - 16t^2 = 0$$

$$16t(2 - t) = 0$$

$$t = 0, \quad t = 2$$

speed at $t=2$

$$|\vec{v}(t)| = \sqrt{32^2 + (32 - 32t)^2}$$

$$|\vec{v}(2)| = \sqrt{32^2 + (32 - 64)^2}$$

$$= \sqrt{32^2 + 32^2}$$

$$= 32\sqrt{2}$$

How far?

$$x(2) = 32 \cdot 2 = 64 \text{ ft}$$

Max height: occurs when y-velocity is zero

$32 - 32t = 0$ or $t = 1$

Max height: $t=1$

$$y(t) = 32t - 16t^2$$

$$y(1) = 32 - 16 = 16 \text{ ft}$$

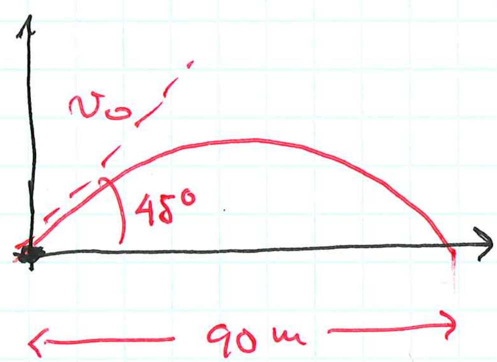
Projectile Motion

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$= m\frac{d}{dt}\vec{v}$$

$$= m\vec{a}$$

Projectile problem



$$\vec{r}''(t) = -9.8\hat{j}$$

$$\vec{r}'(0) = v_0\cos(45°)\hat{i} + v_0\sin(45°)\hat{j}$$

$$\vec{r}(0) = 0\hat{i} + 0\hat{j}$$

$$\vec{v}(t) = \int_0^t \vec{r}''(u)\,du + \vec{v}(0)$$

$$= \int_0^t \langle 0, -9.8\rangle\,du + v_0\frac{\sqrt{2}}{2}(\hat{i}+\hat{j})$$

$$v(t) = \langle 0, -9.8t \rangle + v_0 \frac{\sqrt{2}}{2} \langle 1, 1 \rangle$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(u)\, du$$

$$= \langle 0, 0 \rangle + \int_0^t \langle v_0 \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} - 9.8u \rangle\, du$$

$$= \langle 0, 0 \rangle + \langle \underline{v_0 \frac{\sqrt{2}}{2} t}, v_0 \cdot \frac{\sqrt{2}}{2} t - 4.9t^2 \rangle$$

To find v_0:

Time of landing:

$$v_0 \frac{\sqrt{2}}{2} t_i - 4.9 t_i^2 = 0$$

$$t_i \left(v_0 \frac{\sqrt{2}}{2} - 4.9 t_i \right) = 0$$

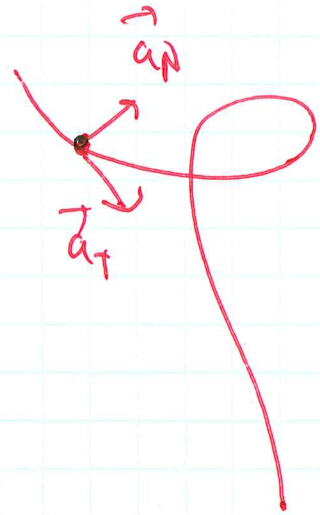
- $y = 0$ at time of impact $\quad 4.9\, t_i = v_0 \frac{\sqrt{2}}{2} \quad -(1)$

- At time of impact $x = 90$ m $\quad v_0 \frac{\sqrt{2}}{2} t_i = 90 \text{ m} \quad -(2)$

From (1) $\quad t_i = \dfrac{v_0 \frac{\sqrt{2}}{2}}{4.9}$

Substitute into (2): $\quad v_0 \frac{\sqrt{2}}{2} \cdot \dfrac{v_0 \frac{\sqrt{2}}{2}}{4.9} = 90 \text{ m}$

$$\frac{1}{2} v_0^2 / 4.9 = 90 \text{ m}$$



$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{v(t)}$$

$$\kappa v(t) = |\vec{T}'(t)|$$

$$\vec{v}(t) = v(t)\,\vec{T}(t)$$

$$\frac{d}{dt}\vec{v}(t) = \frac{d}{dt}\left(v(t)\,\vec{T}(t)\right)$$

$$= v'(t)\,\vec{T}(t) + v(t)\,\vec{T}'(t)$$

$$= v'(t)\,\vec{T}(t) + v(t)\,\underbrace{|\vec{T}'(t)|}_{\kappa}\cdot\vec{N}(t)$$

$$= \underbrace{v'(t)}_{\vec{a}_T}\,\vec{T}(t) + \underbrace{\kappa(t)\,v(t)^2}_{\vec{a}_N}\,\vec{N}(t)$$

Components of Acceleration

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$$

$$\vec{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$$

$$|\vec{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\vec{T}(t) = \frac{1}{\sqrt{2}} \langle -\sin t, \cos t, 1 \rangle$$

$$\vec{T}'(t) = = \frac{1}{\sqrt{2}} \langle -\cos t, -\sin t, 0 \rangle$$

To find $\kappa(t)$:

$$\vec{r}'(t) = \langle -\sin t, \cos t, 1 \rangle$$

$$\vec{r}''(t) = \langle -\cos t, -\sin t, 0 \rangle$$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin t & \cos t & 1 \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$= -\sin t\, \hat{i} + \cos t\, \hat{j} + \hat{k}$$

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} = \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{1}{2}$$

$$\vec{a}(t) = \vec{r}''(t) = \langle -\cos t, -\sin t, 0 \rangle = \vec{N}$$

$$\vec{a} = v' \vec{T} + \kappa v^2 \vec{N}$$

$$= 0 \cdot \vec{T} + \frac{1}{2} (\sqrt{2})^2 \vec{N} = \vec{N}$$

$v = \sqrt{2}$

$v' = 0$