

①

$$f(x, y) = 2x^2 + y^2 - 5y$$

$$f_x(x, y) = 4x$$

$$f_y(x, y) = 2y - 5$$

$$f_x(1, 2) = \underline{4}$$

$$f_y(1, 2) = \underline{\underline{-1}}$$

$$z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$(a, b) = (1, 2)$$

$$f(a, b) = 2 \cdot 1^2 + 2^2 - 5 \cdot 2 = \underline{\underline{-4}}$$

$$z - \underline{\underline{-4}} = \underline{4}(x - \underset{\uparrow}{\underline{a}}) + \underline{\underline{-1}}(y - \underset{\uparrow}{\underline{b}})$$

$$z + 4 = 4x - 4 - y + 2$$

$$z - 4x + y + 4 = 2 - 4$$

$$z - 4x + y = 2 - 8 = -6$$

$$z = e^{x-y}$$

$$f(x, y) = e^{x-y}$$

a b
" "

Tgt plane at $(2, 2, \underline{1})$

$$f_x(x, y) = e^{x-y}$$

$$f_y(x, y) = e^{x-y}(-1)$$

$$f_x(2, 2) = 1$$

$$f_y(2, 2) = -1$$

$$z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$z - \underline{1} = 1(x - 2) + (-1)(y - 2)$$

(3)

Linear approximation to $f(x, y)$ near (a, b)

$$L(x, y) = f(a, b) + \cancel{f_x(a, b)(x-a)} + f_x(a, b)(x-a) + f_y(a, b)(y-b)$$

$$\textcircled{1} \quad (a, b) = (0, 0)$$

$$f(x, y) = e^x \cos(xy)$$

$$f(0, 0) = e^0 \cos(0) = 1$$

$$\underline{f_x(x, y)} = e^x \cos(xy) + e^x (-\sin(xy) \cdot y)$$

$$\underline{f_y(x, y)} = e^x (-\sin(xy) \cdot x)$$

$$\underline{f_x(0, 0)} = e^0 \cos 0 + e^0 (-\cancel{\sin(0)} \cdot 0) = 1$$

$$\underline{f_y(0, 0)} = e^0 (-\sin(0) \cdot 0) = 0$$

$$L(x, y) = 1 + 1 \cdot (x-0) + 0 \cdot (y-0) = 1+x$$

4

2

$$L(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b)$$

$$(a, b) = (2, 5)$$

$$f(a, b) = 6$$

$$f_x(a, b) = 1$$

$$f_y(a, b) = -1$$

$$L(x, y) = 6 + 1(x-2) + (-1)(y-5)$$

$$L(2.2, 4.9) = 6 + 1(2.2-2) + (-1)(4.9-5)$$

$$= 6 + 0.2 + 0.1$$

$$= 6.3$$

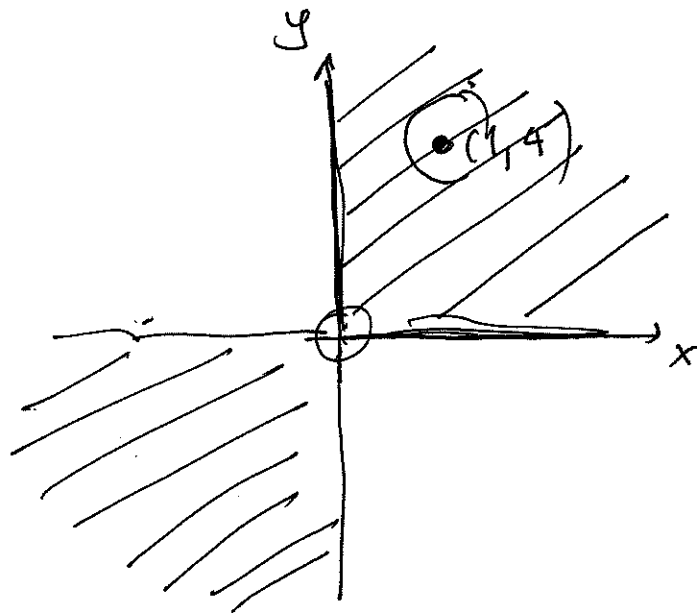
5

$$f(x,y) = \sqrt{xy} = (xy)^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}}$$

$$f_x(x,y) = \frac{1}{2} \cdot x^{-\frac{1}{2}} \cdot y^{\frac{1}{2}}$$

$$f_y(x,y) = \frac{1}{2} \cdot x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}}$$

Domain:



$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$(a,b) = (1,4)$$

$$f(1,4) = 2$$

$$f_x(1,4) = \frac{1}{2} \cdot 1^{-\frac{1}{2}} \cdot 4^{\frac{1}{2}} = 1$$

$$f_y(1,4) = \frac{1}{2} \cdot 1^{\frac{1}{2}} \cdot 4^{-\frac{1}{2}} = \frac{1}{4}$$

$$L(x,y) = 2 + \frac{1}{2}(x-1) + \frac{1}{4}(y-4)$$

$$f(x,y) = \frac{xy}{x^2+y^2} \quad f(0,0) = 0$$

$$f_x(0,0) = \lim_{h \rightarrow 0} \frac{f(h,0) - \cancel{f(0,0)}}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{1}{h} \cdot \frac{h \cdot 0}{h^2 + 0^2} \right) =$$

$$= \lim_{h \rightarrow 0} 0 = 0$$

$$f_y(0,0) = \lim_{h \rightarrow 0} \frac{\cancel{f(0,h)} - \cancel{f(0,0)}}{h}$$

$$= 0$$

$$f_x(x,y) = \frac{y \cdot (x^2+y^2) - xy(2x)}{(x^2+y^2)^2}$$

$$= \frac{yx^2 + y^3 - 2x^2y}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{x^3 - y^2x}{(x^2+y^2)^2}$$

Is $f_x(x,y)$ continuous at $(0,0)$?

$$\lim_{(x,y) \rightarrow (0,0)} \left(\frac{y^3 - x^2y}{(x^2+y^2)^2} \right) \quad \text{DNE}$$

$f_x(x,y)$