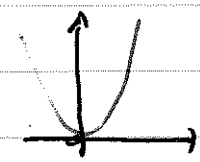
10/8/2018 ①

Maximum and Minimum Values, I

2^{nd} Derivative Test - 1 variable

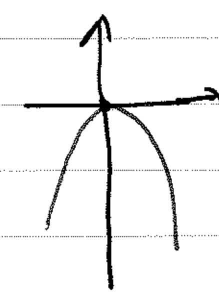
① $f(x) = x^2$



$f'(x) = 2x$

$f''(x) = 2$ $\qquad f''(0) > 0$

② $f(x) = -x^2$



$f'(x) = -2x$

$f''(x) = -2$

$f''(0) = -2 \quad \leftarrow \quad f''(0) < 0$

2nd Derivative Test - 2 variables

① $f(x,y) = x^2+y^2$

$f_x(x,y) = 2x$ $\qquad f_y(x,y) = 2y$

$f_{xx}(x,y) = 2$ $\qquad f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 0$

$$\text{Hess}(f)(0,\cancel{0}) = \begin{pmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$D = +4$

② $f(x,y) = -(x^2+y^2)$

$$\text{Hess}(f) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

3 $f(x,y) = x^2 - y^2$

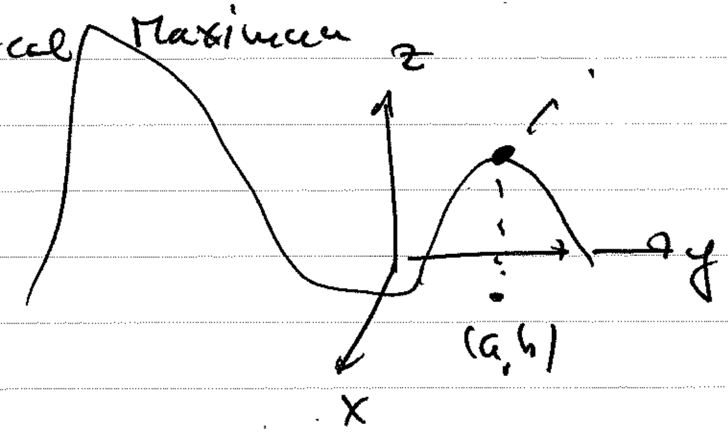
$f_x(x,y) = 2x$ $\qquad$ $f_y(x,y) = -2y$

$f_{xx}(x,y) = 2$ $\qquad$ $f_{yy}(x,y) = -2$

$f_{xy}(x,y) = 0$

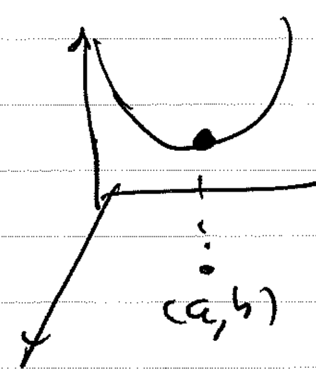
$$\text{Hess}(f)\,(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

Local Maximum



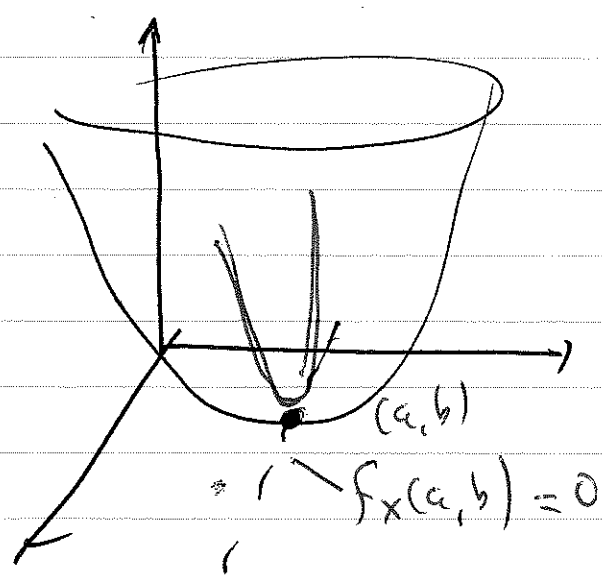
local max

Local Minimum



A function $f(x,y)$ has an <u>absolute maximum</u> at (a,b) if $f(a,b) \geq f(x,y)$ for every (x,y) in the domain of f

A function $f(x,y)$ has an absolute minimum at (a,b) if $f(a,b) \leq f(x,y)$ for every (x,y) in the domain of f



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1 $f(x,y) = x^3 - 3x + 3xy^2$

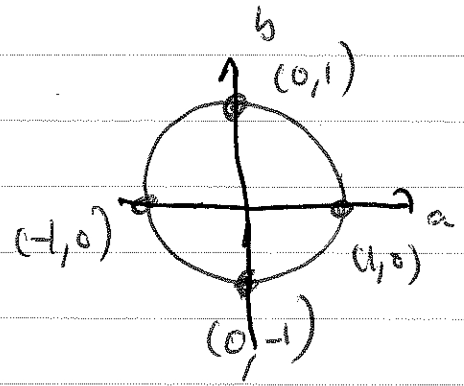
$f_x(x,y) = 3x^2 - 3 + 3y^2$

$f_y(x,y) = 6xy$

(a,b) is a critical point if:

(1) – $3a^2 - 3 + 3b^2 = 0 \Rightarrow 3a^2 + 3b^2 = 3 \Rightarrow a^2 + b^2 = 1$

(2) – $6ab = 0$



e $f(x,y) = x^2 + y^4 + 2xy$

$f_x(x,y) = 2x + 2y$

$f_y(x,y) = 4y^3 + 2x$

(1) $2a + 2b = 0$ $\quad 2a = -2b$

(2) $4b^3 + 2a = 0$ $\quad 4b^3 - 2b = 0$

$2b(2b^2 - 1) = 0$

$\Rightarrow b = 0, \; b = \pm\frac{1}{\sqrt{2}}$

$a = -b$

$\Downarrow$

$(0,0), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

3 $f_x = e^x \cos y$

$f_y = -e^x \sin y$

~~$e^a \cos y = 0$~~

$$\begin{cases} e^a \cos b = 0 \\ e^a \sin b = 0 \end{cases}$$

No critical pts!

2^{nd} Der Test

1 $f(x,y) = x^2 + y^2$ Local min

$$\text{Hess}(f) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{pmatrix}$$

2 $f(x,y) = -(x^2+y^2)$ Local max

$$\text{Hess}(f) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

① $f(x,y) = x^3 - 3x + 3xy^2$

$f_x = 3x^2 - 3 + 3y^2 \qquad f_{xy} = 6y$

$f_{xx} = 6x$

$f_y = 6xy$

$f_{yy} = 6x$

$$\text{Hess}(f) = \begin{pmatrix} 6x & 6y \\ 6y & 6x \end{pmatrix}$$

$D = 36(x^2 - y^2)$

② $f(x,y) = x^2 + y^4 + 2xy$

$f_x = 2x + 2y \qquad f_{xy} = 2$

$f_{xx} = 2$

$f_y = 4y^3 + 2x$

$f_{yy} = 12y^2$

$$\text{Hess}(f) = \begin{pmatrix} 2 & 2 \\ 2 & 12y^2 \end{pmatrix}$$

$D = 24y^2 - 4$