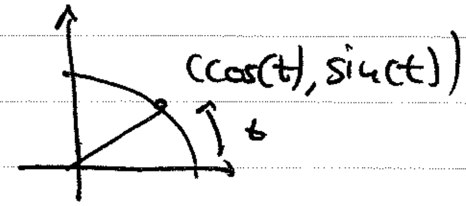
10/10/18 1

1) Parameterize the circle:

$x(t) = \cos(t)$

$y(t) = \sin(t)$



$0 \leq t \leq 2\pi$

$$\phi(t) = f(x(t), y(t)) = \overset{x^2}{\cos^2(t)} - \overset{y^2}{\sin^2(t)}$$

So, to find the max/min of f on the circle, find the max/min of the function

$$\phi(t) = \cos^2 t - \sin^2 t \qquad 0 \leq t \leq 2\pi$$

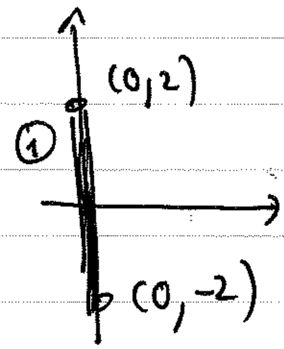
1) $\phi'(t) = -2\cos(t)\sin(t) - 2\sin(t)\cos(t)$
$= -4\sin(t)\cos(t)$

t	$\phi(t)$
0	+1
$\pi/2$	−1
π	+1
$3\pi/2$	−1
2π	+1

Abs max: +1

Abs min: −1

2) ① $f(x,y) = x^2+y^2-2x$



$x(t) = 0$

$y(t) = t \quad -2 \le t \le 2$

$f(x(t), y(t)) = 0 + t^2 - 2\cdot 0 = t^2$

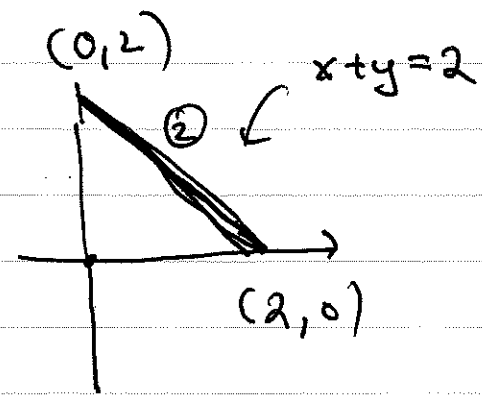
$\phi_1(t) = t^2 \qquad -2 \le t \le 2$

$\phi_1'(t) = 2t \qquad \phi_1'(t) = 0$ at $t=0$

t	$\phi_1(t)$
-2	4
0	0
2	4

Min: 0

Max: 4



②

$x(t) = t$

$y(t) = 2-t$

$0 \le t \le 2$

$\phi_2(t) = t^2 + (2-t)^2 - 2t \qquad 0 \le t \le 2$

$\phi_2'(t) = 2t + 2(2-t)(-1) - 2$

$= 2t + 2(t-2) - 2$

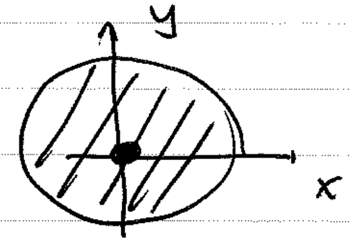
$= 4t - 6 = 0$ if $t = 3/2$

t	$\phi_2(t)$
0	4
3/2	something
2	0

Finding Extreme Values

$f(x,y) = x^2 - y^2$ on $D = \{(x,y) : x^2+y^2 \le 1\}$

Recall from p. 1



on the bdry: we found that extreme values were ± 1

Interior critical pts:

$f_x(x,y) = 2x$

$f_y(x,y) = -2y$

only critical pt: $(x,y) = (0,0)$

$f_{xx} = 2$

$f_{xy} = 0$

$f_{yy} = -2$

$\mathrm{Hess}(f) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$

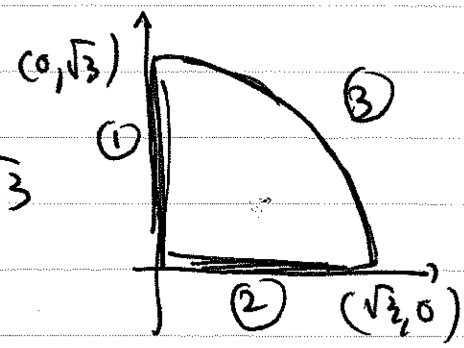
$D = -4$

Max: $+1$

Min: -1

$f(x,y) = xy^2$

on $D = \{(x,y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$



(1) : $x(t) = 0$

$y(t) = t \quad 0 \leq t \leq \sqrt{3}$

$\phi_1(t) = f(x(t), y(t))$

$= 0$

(2) : $x(t) = t \quad 0 \leq t \leq \sqrt{3}$

$y(t) = 0$

$\phi_2(t) = f(x(t), y(t)) = 0$

(3) $x(t) = \sqrt{3} \cos t \quad 0 \leq t \leq \frac{\pi}{2}$

$y(t) = \sqrt{3} \sin t$

$\phi_3(t) = f(x(t), y(t)) = \sqrt{3} \cos t \, (\sqrt{3} \sin t)^2$

$= 3\sqrt{3} \cos t \cdot \sin^2 t$

$$\phi_3'(t) = 3\sqrt{3}\left[-\sin^3 t + \cos t \cdot 2\,\overset{\sin}{\cancel{\cos}}\,t \cdot \cos t\right]$$

$$= 3\sqrt{3}\left[-\sin^3 t + 2\cos^2 t \sin t\right]$$

$\sin^2 + \cos^2 = 1$

$$= 3\sqrt{3}\,\sin t\left[\underbrace{-\sin^2 t} + 2\cos^2 t\right]$$

$\cos^2 - 1 = -\sin^2$

$$= 3\sqrt{3}\,\sin t\left[\cos^2 t - 1 + 2\cos^2 t\right]$$

$$= 3\sqrt{3}\,\sin t\left[3\cos^2 t - 1\right]$$

$\phi_3'(t) = 0$ if $\cos^2 t_0 = \frac{1}{3}$

$\sin^2 t_0 = \frac{2}{3}$

$$\phi_3(t_0) = 3\sqrt{3}\,\cos t_0 \cdot \sin^2 t_0$$

$$= 3\cancel{\sqrt{3}}\,\frac{1}{\cancel{\sqrt{3}}} \cdot \frac{2}{3} = 2$$

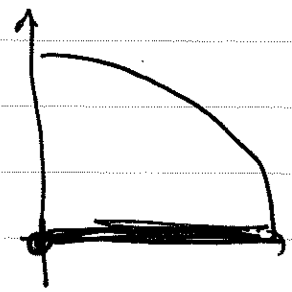
t	$\phi_3(t)$
0	0
t_0	2
$\frac{\pi}{2}$	0

- $f(x,y) = xy^2$

$f_x(x,y) = \cancel{x}\, y^2 \Rightarrow y = 0$

$f_y(x,y) = 2xy \Rightarrow x = \text{anything}$

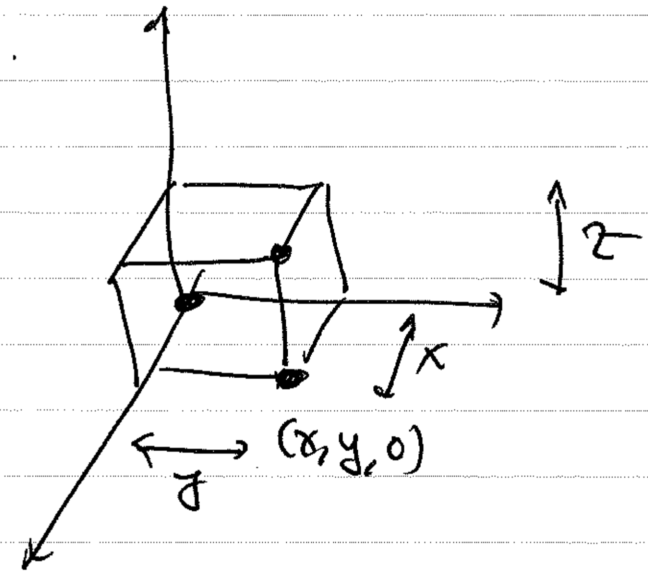
No interior critical pts!



Conclusion: Abs max: 2

Abs min: 0

Word Problem



$V = xyz$

solve for z in

$x + 2y + 3z = 6$

$3z = 6 - x - 2y$

$z = 2 - \frac{1}{3}x - \frac{2}{3}y$

$V(x,y) = xy\left(2 - \frac{1}{3}x - \frac{2}{3}y\right) = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2$

$V_x(x,y) = 2y - \frac{2}{3}xy - \frac{2}{3}y^2 = 0$

$V_y(x,y) = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy = 0$

$V_x(x,y) = y\left(2 - \frac{2}{3}x - \frac{2}{3}y\right) = 0$

$V_y(x,y) = x\left(2 - \frac{1}{3}x - \frac{4}{3}y\right) = 0$

10/10/18 ⑨

CP: $2 - \frac{2}{3}x - \frac{2}{3}y = 0$

$$2 - \frac{1}{3}x - \frac{4}{3}y = 0$$

$$6 - 2x - \cancel{4}\overset{2}{}y = 0$$

$$6 - \cancel{2}x - 4y = 0$$

$$\begin{cases} 2x + 2y = 6 \\ x + 4y = 6 \end{cases}$$

$$\begin{matrix} ① \\ ② \end{matrix} \begin{cases} 2x + 2y = 6 \\ 2x + 8y = 12 \end{cases}$$

② − ①: $0 + 6y = 6$

$$\boxed{\begin{matrix} y = 1 \\ x = 2 \end{matrix}}$$