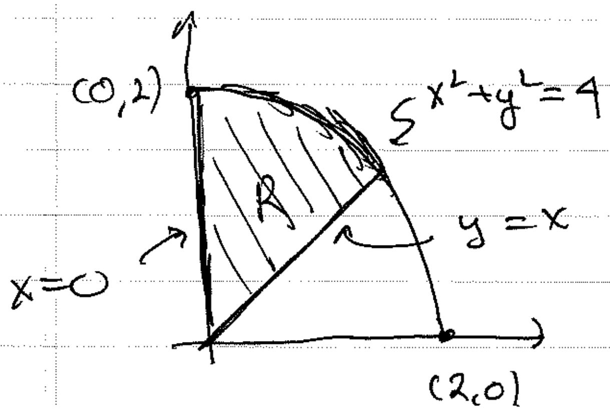


$0 \leq r \leq 2$

$\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

$a=0 \quad b=2$

$\alpha = \frac{\pi}{4} \quad \beta = \frac{\pi}{2}$

$f(x,y) = 2x - y$

$f(r\cos\theta, r\sin\theta) = 2r\cos\theta - r\sin\theta$

$\iint_R f(x,y)\, dA =$

$$\int_{\pi/4}^{\pi/2} \left[\int_0^2 (2r\cos\theta - r\sin\theta)\ \underline{r}\ dr \right] d\theta$$

$$\int_0^2 (2r\cos\theta - r\sin\theta)\underline{r}\,dr$$

$$= \int_0^2 (2r^2\cos\theta - r^2\sin\theta)\,dr$$

$$= \cos\theta\left(\int_0^2 2r^2\,dr^2\right) - \sin\theta\left(\int_0^2 r^2\,dr\right)$$

$$= \cos\theta \left[\frac{2r^3}{3}\right]\Big|_0^2 - \sin\theta \left[\frac{r^3}{3}\right]\Big|_0^2$$

$$= \frac{16}{3}\cos\theta - \frac{8}{3}\sin\theta$$

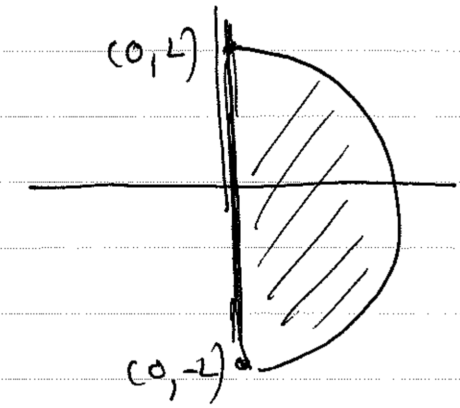
$$\int_{\pi/4}^{\pi/2} \left(\frac{16}{3}\cos\theta - \frac{8}{3}\sin\theta\right) d\theta =$$

$$\frac{16}{3}\left[\sin\theta\right]\Big|_{\pi/4}^{\pi/2} - \frac{8}{3}\left[-\cos\theta\right]\Big|_{\pi/4}^{\pi/2} =$$

$$\frac{16}{3}\left(1 - \frac{\sqrt{2}}{2}\right) - \frac{8}{3}\left(-0 + \frac{1}{\sqrt{2}}\right) =$$

$$\frac{16}{3} - \frac{16}{3}\frac{\sqrt{2}}{2} - \frac{8}{3}\frac{1}{\sqrt{2}}$$

$$\iint_R e^{-(x^2+y^2)}\, dA$$



$$\boxed{\begin{aligned} &0 \le r \le 2 \\ &-\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2} \end{aligned}} \quad R$$

$x = \sqrt{4-y^2}$

$x^2 = 4-y^2$

$x^2+y^2 = 4$

In polar coordinates:

$x^2+y^2 = r^2$

so $e^{-((r\cos\theta)^2+(r\sin\theta)^2)} = \boxed{e^{-r^2}}$

$$\iint_R e^{-(x^2+y^2)}\, dA = \int_{-\pi/2}^{\pi/2}\left(\int_0^2 e^{-r^2}\, r\, dr\right) d\theta$$

$u = r^2$

$du = 2r\,dr$

$r=0 \Rightarrow u=0$

$r=2 \Rightarrow u=4$

$$= \int_{-\pi/2}^{\pi/2}\left(\int_0^4 e^{-u}\ \tfrac{1}{2}\, du\right) d\theta$$

$$= \int_{-\pi/2}^{\pi/4}\left(\tfrac{1}{2}\left[-e^{-u}\right]\Big|_0^4\right) d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \tfrac{1}{2}(1-e^{-4})\, d\theta$$

$$= \tfrac{1}{2}(1-e^{-4})\left(\tfrac{\pi}{2}-\left(-\tfrac{\pi}{2}\right)\right) = \boxed{\tfrac{\pi}{2}(1-e^{-4})}$$

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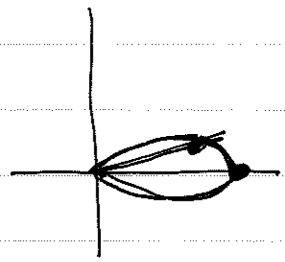
$$A(R) = \iint_R 1 \cdot dA$$



$$-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$$

$$0 \le r \le \cancel{3\cos\theta}\ \cos 3\theta$$

θ	r
0	1
$\frac{\pi}{12}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{6}$	0

$$A(R) = \int_{-\pi/6}^{\pi/6} \int_0^{\cos 3\theta} r\, dr\, d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \left(\left[\frac{r^2}{2} \right] \Big|_0^{\cos 3\theta} \right) d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \frac{\cos^2 3\theta}{2}\, d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}\left(\frac{1+\cos 6\theta}{2} \right) d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \left(\frac{1}{4} + \frac{1}{4}\cos 6\theta \right) d\theta$$

$$= \frac{1}{4} \cdot \frac{\pi}{3} + \left[\frac{1}{24} \sin 6\theta \right] \Big|_{-\pi/6}^{\pi/6}$$

$$= \frac{\pi}{12} + \frac{1}{24}\left[0 - 0 \right]$$

D: $0 \le r \le 5$ $\quad$ $0 \le \theta \le 2\pi$

$$f(r\cos\theta, r\sin\theta) = r^2$$

$$\iint_D z \, dA = \int_0^{2\pi} \int_0^5 \underline{r^2} \, \underline{r \, dr \, d\theta}$$

$$= \int_0^{2\pi} \left[\int_0^5 r^3 \, dr \right] d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^4}{4} \right] \Bigg|_0^5 \, d\theta$$

$$= \int_0^{2\pi} \frac{125}{4} \, d\theta$$

$$= 2\pi \cdot \frac{125}{4}$$

$$= \frac{125\pi}{2}$$