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Math 213 - Surface Area

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October 29, 2018

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Homework

- Re-read section 15.5
- Begin work on problems 1-11 (odd) in section 15.5
- Read section 15.6 for Wednesday October 31
- Work on Webwork C2

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Unit III: Multiple Integrals

- Lecture 24 Double Integrals over Rectangles
- Lecture 25 Double Integrals over General Regions
- Lecture 26 Double Integrals in Polar Coodinates
- Lecture 27 Applications of Double Integrals
- Lecture 28 Surface Area
- Lecture 29 Triple Integrals
- Lecture 30 Triple Integrals in Cylindrical Coordinates
- Lecture 31 Triple Integrals in Spherical Coordinates
- Lecture 32 Change of Variable in Multiple Integrals, Part I
- Lecture 33 Change of Variable in Multiple Integrals, Part II
- Lecture 34 Exam III Review

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Goals of the Day

- Understand surface area as a limit of Riemann sums
- Understand how to compute surface area as a double integral



Suppose S is a surface in \mathbb{R}^3 over a rectangle R.

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Over each rectangle R_{ij} in the xy plane



Suppose S is a surface in \mathbb{R}^3 over a rectangle R.

Over each rectangle R_{ij} in the xy plane is a piece of surface, T_{ij} , of area ΔS_{ij}

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- Suppose S is a surface in \mathbb{R}^3 over a rectangle R.
- Over each rectangle R_{ij} in the xy plane is a piece of surface, T_{ij} , of area ΔS_{ij}
- We'll estimate the area of this piece of surface by using the tangent plane to S

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Take the linear approximation from the midpoint:

$$L(x, y) = f(x_i^*, y_i^*) + f_x(x_i^*, y_i^*)(x - x_i^*) + f_y(x_i^*, y_i^*)(y - y_i^*)$$



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Find the vectors **a** and that span the cyan rectangle, and compute the area $A = |\mathbf{a} \times \mathbf{b}|$



$$f_x(x_i^*, y_i^*) = a, \quad f_y(x_i^*, y_i^*) = b$$

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The parallelogram is spanned by the vectors

$$\mathbf{a} = (\Delta x, \mathbf{0}, a\Delta x)$$
$$\mathbf{b} = (\mathbf{0}, \Delta y, b\Delta y)$$
$$\mathbf{a} \times \mathbf{b} = (-a\mathbf{i} - b\mathbf{j} + \mathbf{k}) \Delta x \Delta y$$

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The parallelogram has area

$$\Delta A = |\mathbf{a} imes \mathbf{b}|$$

= $\sqrt{1 + a^2 + b^2} \Delta x \Delta y$

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If $R = [a, b] \times [c, d]$, then, the surface area of S is given by

$$\int_{a}^{b} \int_{c}^{d} \sqrt{1 + f_{x}(x, y)^{2} + f_{y}(x, y)^{2}} \, dx \, dy$$

For a general region R, the area of a surface over R is

$$\iint_{R} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^{2} + \left(\frac{\partial z}{\partial y}\right)^{2}} \, dA$$

... so any method of evaluating this double integral (type I, type II, polar) is fair game!

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$$\iint_{R} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^{2} + \left(\frac{\partial z}{\partial y}\right)^{2}} \, dA$$

Find the surface area of ...

- 1. The part of the plane 5x + 3y z + 6 = 0 above the rectangle $[1, 4] \times [2, 6]$
- 2. The part of the surface $2y + 4z = x^2 = 5$ above the triangle with vertices (0, 0), (2, 0) and (2, 4)
- 3. The part of the hyperbolic paraboloid $z = y^2 x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$
- 4. The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane z = 1

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