10/31/2018

$$\iiint_B (xy + z^2)\, dV = \qquad B = \underset{x}{[0,2]} \times \underset{y}{[0,1]} \times \underset{z}{[0,3]}$$

$$\underset{z}{\overset{3}{\int_0}} \left(\underset{y}{\overset{1}{\int_0}} \left(\underset{x}{\overset{2}{\int_0}} (xy + z^2)\, dx \right) dy \right) dz$$

③ ② ①

① $\int_0^2 (xy + z^2)\, dx = y \int_0^2 x\, dx + z^2 \int_0^2 dx$

$= y \left[\frac{x^2}{2}\right]\Big|_0^2 + 2z^2$

$= 2y + 2z^2$

② $\int_0^1 (2y + 2z^2)\, dy = \int_0^1 2y\, dy + 2z^2 \int_0^1 dy$

$= [y^2]\Big|_0^1 + 2z^2 \cdot 1$

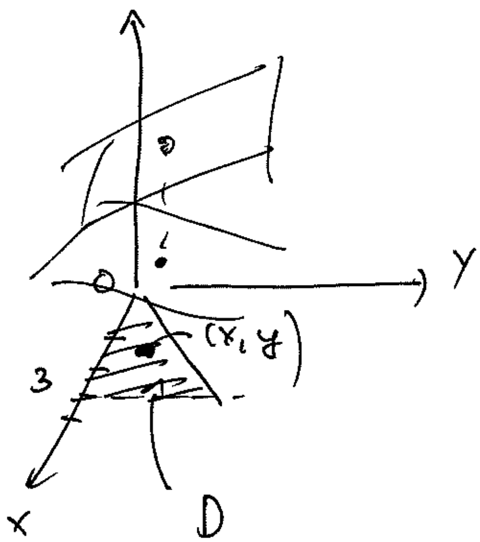
$= 1 + 2z^2$

③ $\int_0^3 (1 + 2z^2)\, dz = \left[z + \frac{2}{3} z^3\right]\Big|_0^3$

$= 3 + \frac{2}{3} \cdot 3^3 = 3 + 2 \cdot 9 = \boxed{21}$

$$\iiint_E y \, dV$$

$$E = \{ (x, y, z) : \quad (x,y) \in D, \quad x - y \le z \le x + y \}$$



$u_1(x,y) = x - y$

$u_2(x,y) = x + y$

$D = \quad 0 \le x \le 3, \quad 0 \le y \le x$

$$\iiint_E y \, dV = \iint_D \left(\int_{x-y}^{x+y} y \, dz \right) dA$$

$$= \iint_D \left(y \left(\int_{x-y}^{x+y} 1 \, dz \right) \right) dA$$

$$= \iint_D \left(y \cdot [z] \Big|_{x-y}^{x+y} \right) dA$$

$$= \iint_D y \cdot [(x+y) - (x-y)] \, dA$$

$$= \iint_D y \cdot 2y \; dA$$

$$= \iint_D 2y^2 \; dA$$

$$D = \{(x,y): \quad 0 \le x \le 3, \qquad 0 \le y \le x\}$$

$$\iint_D 2y^2 \, dA = \int_0^3 \left(\int_0^x 2y^2 \, dy \right) dx$$

$$= \int_0^3 \left(\left[\tfrac{2}{3} y^3\right]\Big|_0^x \right) dx$$

$$= \int_0^3 \left(\tfrac{2}{3} x^3 \right) dx$$

$$= \left[\frac{2}{4 \cdot 3} x^4 \right]\Big|_0^3$$

$$= \left[\frac{1}{6} x^4 \right]\Big|_0^3$$

$$= \frac{1}{6} \cdot 3^4$$

$$= \frac{1}{2} \cdot 3^3$$

$$= \frac{27}{2}$$

$$\overset{③}{\int_0^1} \left(\overset{②}{\int_y^{2y}} \left(\overset{①}{\int_0^{x+y}} 6xy \, dz \right) dx \right) dy$$

1

$$\boxed{6x^2y + 6xy^2}$$

$$\int_0^{x+y} 6xy \, dz = 6xy \int_0^{x+y} dz$$

$$= 6xy \left(z \Big|_0^{x+y} \right)$$

$$= 6xy(x+y)$$

2

$$\int_y^{2y} (6x^2y + 6xy^2) \, dx =$$

$$= 6y \int_y^{2y} x^2 \, dx + 6y^2 \int_y^{2y} x \, dx$$

$$= 6y \left[\frac{x^3}{3} \right]_y^{2y} + 6y^2 \left[\frac{x^2}{2} \right]_y^{2y}$$

$$= 6y \left[\frac{8y^3}{3} - \frac{y^3}{3} \right] + 6y^2 \left[\frac{4y^2}{2} - \frac{y^2}{2} \right]$$

$$= 6y\left[\frac{7y^3}{3}\right] + 6y^2\left[\frac{3y^2}{2}\right]$$

$$= 2y \cdot 7y^3 + 3y^2 \cdot 3y^2$$

$$= 14y^4 + 9y^4$$

$$\cancel{= 28y^4}$$

$$= 23y^4$$

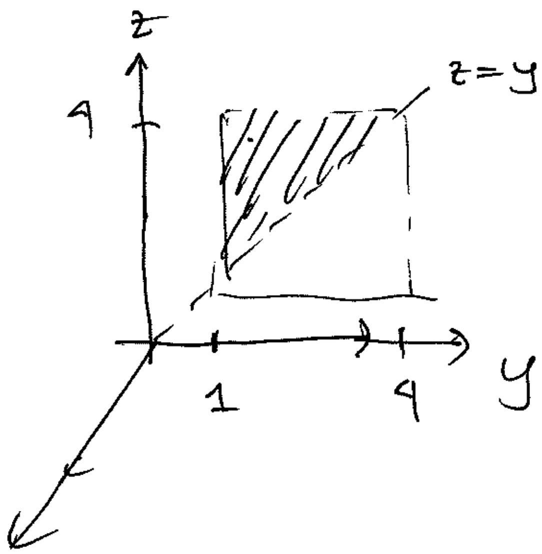
3 $$\int_0^1 23y^4\,dy = 23\left[\frac{y^5}{5}\right]\Big|_0^1$$

$$= \boxed{\frac{23}{5}}$$

$$\iiint_E \frac{z}{x^2+z^2}\, dV$$

$u_1(y,z)$ $u_2(y,z)$

$$E = \{(x,y,z) : \quad 1 \le y \le 4, \quad y \le z \le 4, \quad 0 \le x \le z\}$$

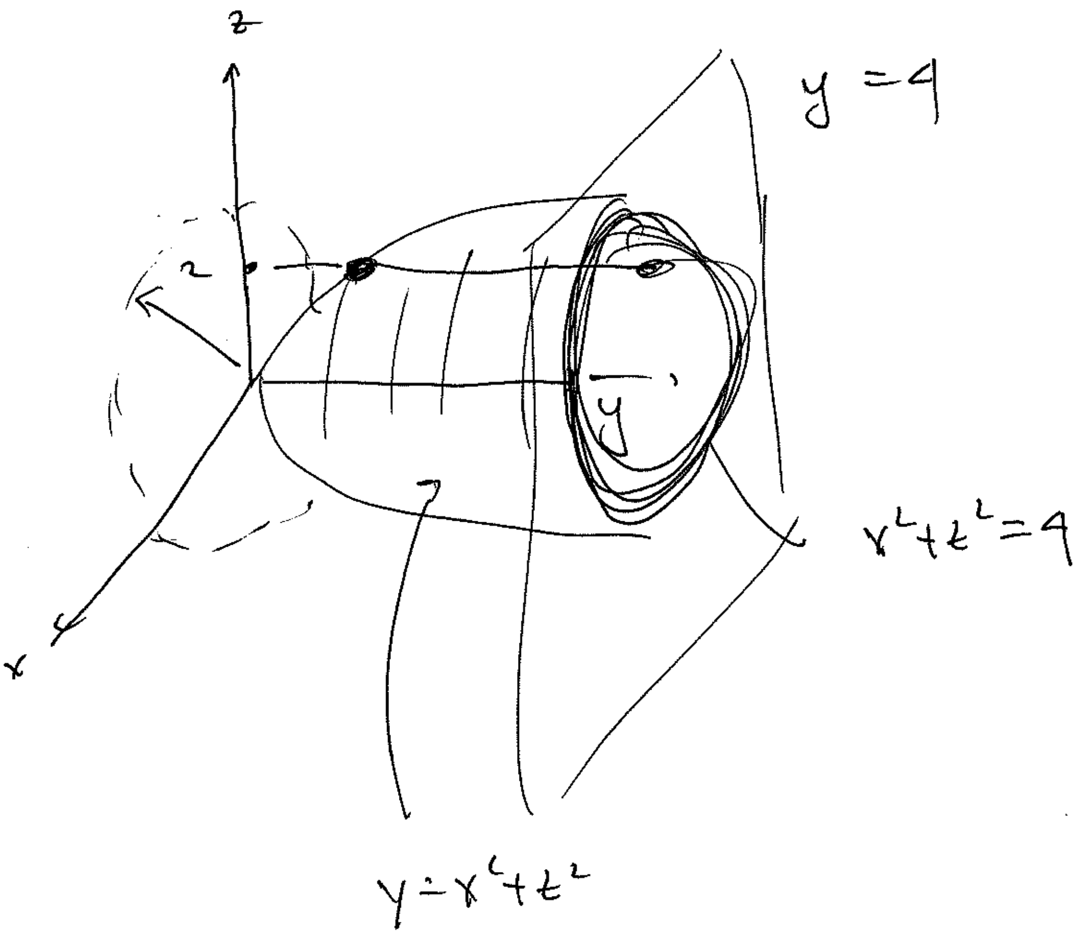


$$D = \{(y,z) : 1 \le y \le 4, \ y \le z \le 4\}$$

$$\iiint_E \frac{z}{x^2+z^2}\, dV = \iint_D \left(\int_0^z \frac{z}{x^2+z^2} \, dx \right) dA$$

$$\iiint_E \sqrt{x^2+z^2}\, dV$$

E bdd by $y = x^2 + z^2$, and $y = 4$



$$D = \{(x,z) : x^2 + z^2 \le 4\}$$

$$u_1(x,z) = x^2 + z^2$$

$$u_2(x,z) = 4$$