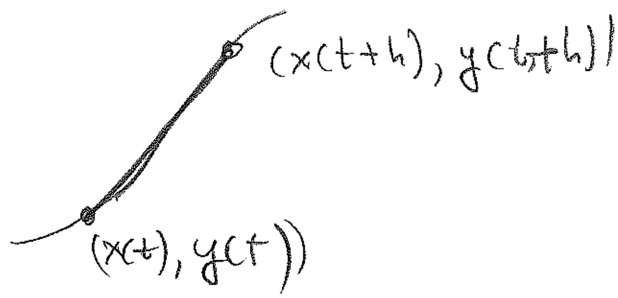
Line Integrals 11/16/2018



$$\Delta x \simeq \frac{dx}{dt} h \qquad h = \Delta t$$

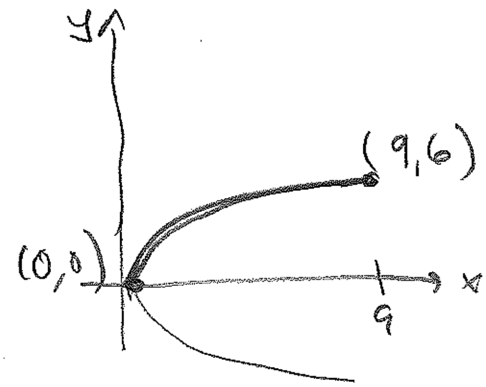
$$\Delta y = \frac{dy}{dt} \cdot h$$

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 \cdot h^2 + \left(\frac{dy}{dt}\right)^2 h^2}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot h$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$$\int_C \left(\frac{x}{y}\right) ds \qquad C: \; x = t^2 \quad y = 2t \quad 0 \le t \le 3$$



$$\frac{x}{y} = \frac{t^2}{2t} = \frac{t}{2}$$

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 2$$

$$t = \frac{y}{2}$$

$$x = \left(\frac{y}{2}\right)^2$$

$$ds = \sqrt{4t^2 + 4}\; dt$$

$$= 2\sqrt{t^2+1}\; dt$$

$$\int_C \left(\frac{x}{y}\right) ds = \int_0^3 \left(\frac{t}{2}\right) \cdot 2\sqrt{t^2+1}\; dt$$

$$= \int_0^3 t \cdot \sqrt{t^2+1}\; dt$$

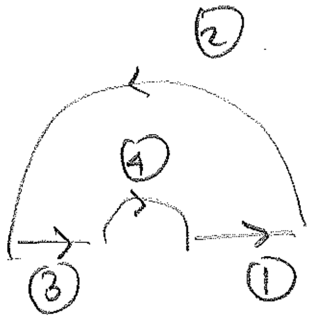
$$u = 1 + t^2$$
$$du = 2t\, dt$$

t	u
0	1
3	10

$$= \frac{1}{2} \int_1^{10} \sqrt{u}\; du$$

$$= \frac{1}{2} \left[\frac{2}{3} u^{3/2}\right]\Big|_1^{10}$$

$$= \frac{1}{3}\left(10^{3/2} - 1\right)$$



$f(x,y) = xy$

① $x(t) = t \qquad 1 \le t \le 2$

$y(t) = 0$

② $x(t) = \cos t \qquad 0 \le t \le \pi$

$y(t) = \sin t$

③ $= 0$

④ $x(t) = \frac{1}{2}\cos t \qquad 0 \le t \le \pi$

$y(t) = \frac{1}{2}\sin t \qquad$ (use − sign)

$$\int_{\curvearrowright} = -\int_{\curvearrowleft}$$

②: $ds = \sqrt{x'(t)^2 + y'(t)^2}\; dt$

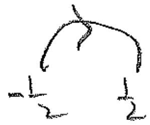
$= \sqrt{(-\sin t)^2 + (\cos(t))^2}\; dt$

$= dt$

$$\int_{②} xy\, ds = \int_0^{\pi} \cos t \sin t\, dt \qquad u = \cos t, \quad du = -\sin t\, dt$$

$$= -\int_1^{-1} u\, du$$

$$= \int_{-1}^{1} u\, du = \left[\frac{u^2}{2}\right]_{-1}^{1} - 1$$



$$\int_{(4)} xy\, ds =$$

$$-\int_0^{\pi} \frac{1}{8}\cos t \sin t\, dt =$$

$$-\frac{1}{8}\cdot 1$$

$x(t) = \frac{1}{2}\cos t$

$y(t) = \frac{1}{2}\sin t$

$ds = \frac{1}{2}\, dt$

$$\int xy\, ds = 1 - \frac{1}{8} = \frac{7}{8}$$

Example: A wire (thin) goes along a curve C and has density $\rho(x,y)$

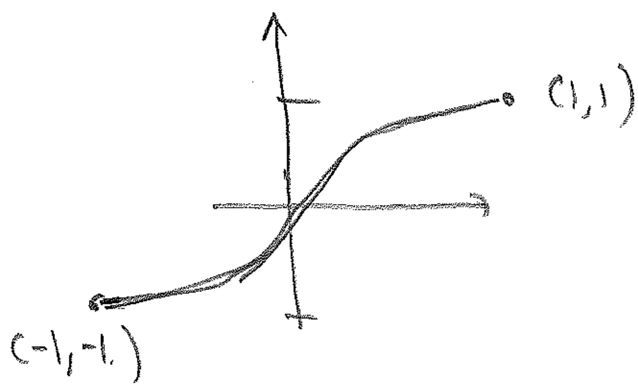
$$M = \int_C \rho(x,y)\, ds$$

1) $\int_C e^x \, dx$ $\qquad$ C: $x = y^3$ from $(-1,-1)$ to $(1,1)$

$x(t) = t^3$

$y(t) = t$

$\underline{-1} \le t \le \underline{+1}$



$dx = x'(t)\, dt = 3t^2 dt$

$$\int_C e^x \, dx = \int_{-1}^{+1} e^{t^3} \, 3t^2 \, dt$$

$u = t^3$

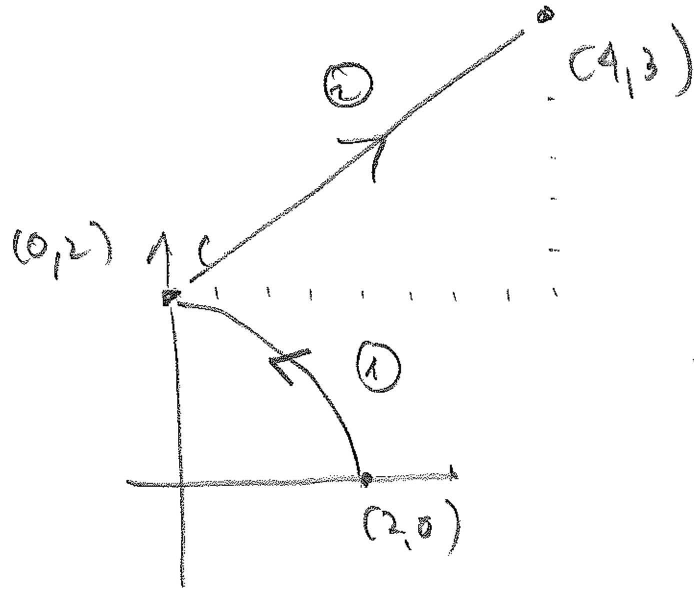
$du = 3t^2 dt$

t	u
-1	-1
1	1

$$= \int_{-1}^{1} e^u \, du$$

$$= e^u \Big|_{-1}^{1}$$

$$= e - e^{-1}$$



Path 1:

$x(t) = 2\cos t \quad 0 \le t \le \frac{\pi}{2}$

$y(t) = 2\sin t$

Path 2:

$$\int_C x^2\,dx = \int_{1} x^2\,dx + \int_{2} x^2\,dx$$

$$\int_C y^2\,dy = \int_{1} y^2\,dy + \int_{2} y^2\,dy$$

Path 2: $(0,2)$ to $(4,3)$

$$\begin{array}{r} (4,3) \\ -(0,2) \\ \hline (4,1) \end{array}$$

$\vec{v} = \langle 4,1 \rangle$

$x(t) = 0 + 4t \qquad 0 \le t \le 1$

$y(t) = 2 + t$

Path 1:

$$x(t) = 2\cos t \qquad 0 \le t \le \frac{\pi}{2}$$

$$y(t) = 2\sin t$$

$$x'(t)\,dt = -2\sin t\,dt$$

$$y'(t)\,dt = 2\cos t\,dt$$

Path 2

$$x(t) = 4t \qquad 0 \le t \le 1$$

$$y(t) = 2 + t$$

$$x'(t)\,dt = 4\,dt$$

$$y'(t)\,dt = dt$$