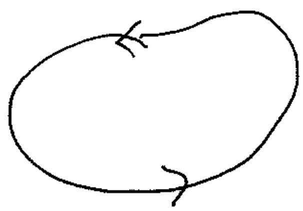
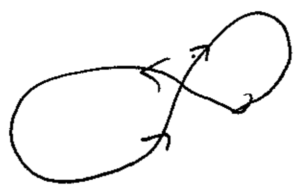
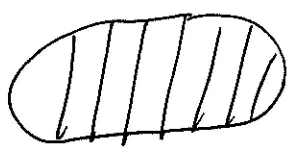


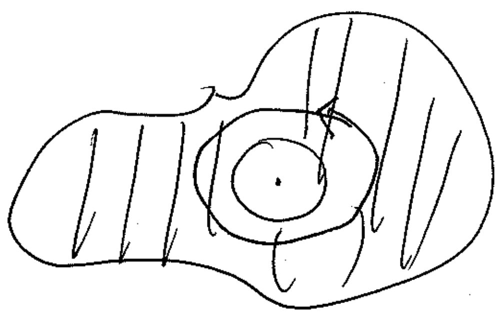
simple closed curve



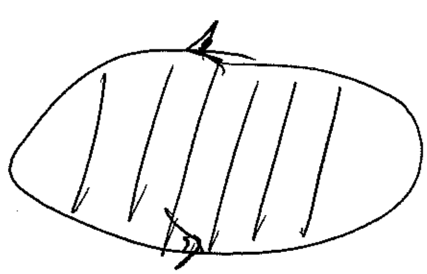
closed but not simple



simply connected (no holes)



not simply connected



positively oriented:

domain to the left

(curve is counterclockwise)

$$\vec{F}(x,y) = P(x,y)\,\vec{i} + Q(x,y)\,\vec{j}$$

$$\int_C P\,dx + Q\,dy = \int_C \vec{F} \cdot d\vec{r}$$

FTC

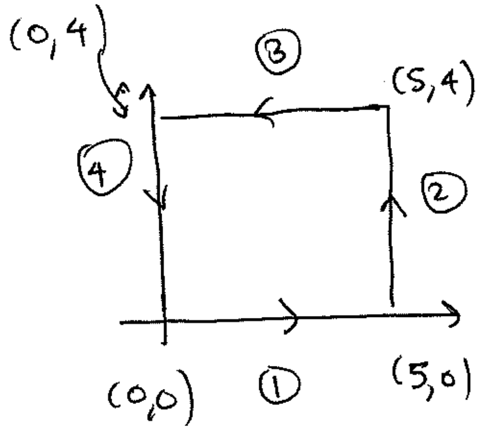
$$\int_a^b f'(x)\,dx$$

$$f(b) - f(a)$$

Green

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$\int_C P\,dx + Q\,dy$$



1 $\langle 0,0\rangle + t\langle 5,0\rangle$

$x(t) = 5t \qquad 0 \le t \le 1$
$y(t) = 0$

2 $\langle 5,0\rangle + t\langle 0,4\rangle$

$x(t) = 5$
$y(t) = 4t \qquad 0 \le t \le 1$

3 $\langle 5,4\rangle + t\langle -5,0\rangle$

$x(t) = 5-5t \qquad 0 \le t \le 1$
$y(t) = 4$

4 $\langle 0,4\rangle + t\langle 0,-4\rangle$

$x(t) = 0$
$y(t) = 4-4t$

$$\oint_C y^2\,dx = \cancel{1} + \cancel{2} + \int_0^1 16(-5)\,dt + \cancel{4} = \boxed{-80}$$

$$\oint_C x^2y\,dy = \cancel{1} + \int_0^1 25(4t)\,4\,dt$$

$$= 400 \cdot \int_0^1 t\,dt = 400 \cdot \frac{t^2}{2}\Big|_0^1 = \boxed{200}$$

$$\cancel{3} + \cancel{4}$$

$$\oint_C y^2\,dx + x^2y\,dy = 120$$

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$$\oint_C \overset{P}{\widetilde{y^2}}\,dx + \overset{Q}{\widetilde{x^2 y}}\,dy$$

$$\frac{\partial Q}{\partial x} = 2xy \qquad \frac{\partial P}{\partial y} = 2y$$

$$\oint_C y^2\,dx + x^2 y\,dy = \iint_R (2xy - 2y)\,dA$$

$$= \int_0^5 \left(\int_0^4 (2xy - 2y)\,dy \right) dx$$

$$= \int_0^5 \left[2x\frac{y^2}{2} - 2\cdot\frac{y^2}{2} \right]\Bigg|_{y=0}^{y=4} dx$$

$$= \int_0^5 \left(\cancel{2}x \cdot \frac{16}{\cancel{2}} - 2\cdot\frac{16}{2} \right) dx$$

$$= \int_0^5 16(x-1)\,dx$$

$$= \left[\overset{8}{\cancel{16}} \cdot \frac{x^2}{\cancel{2}} - 16x \right]\Bigg|_0^5$$

$$= 8\cdot 25 - 16\cdot 5$$

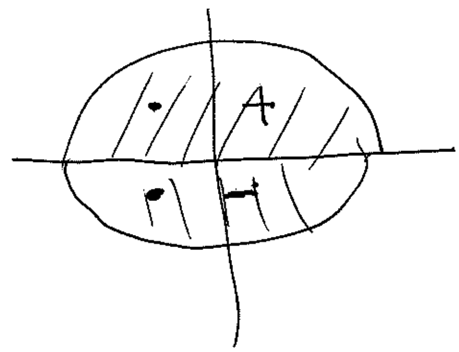
$$= 200 - 80$$

$$= \boxed{120}$$

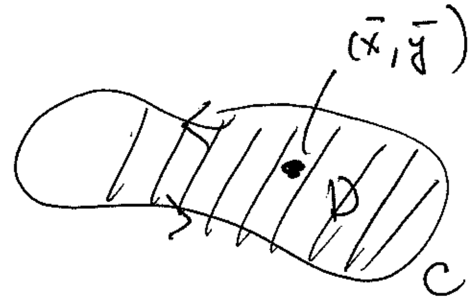
$$\oint_C \underbrace{y^4}_{P}\,dx + \underbrace{2xy^3}_{Q}\,dy \qquad C = \text{ellipse } x^2+2y^2=2$$

$$\frac{\partial P}{\partial y} = 4y^3 \qquad \frac{\partial Q}{\partial x} = 2y^3$$

$$= \iint_D y^3 \, dA$$



$$= 0$$



$$\bar{x} = \frac{1}{A} \iint_D \boxed{x} \, dA$$

$$\bar{y} = \frac{1}{A} \iint_D y \, dA$$

To find a formula for $\bar{x}$ by integrating over C,
find P and Q so that $\frac{\partial Q}{\partial x} - \cancel{\frac{\partial P}{\partial y}} = x$

$$P = 0 \qquad Q(x) = \frac{x^2}{2}$$

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$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D x \, dA$$

$$= \oint_C \left(0 \cdot dx + \frac{x^2}{2} dy\right)$$

$$\iint_D x \, dA = \oint_C \frac{x^2}{2} dy$$

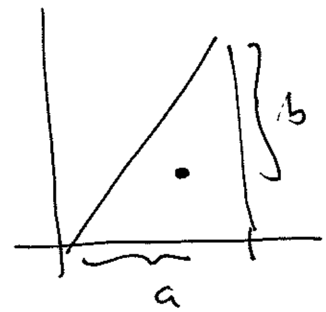
$$\bar{x} = \frac{1}{A} \iint x \, dA = \frac{1}{2A} \oint_C x^2 \, dy$$

$dx = x'(t)\,dt$

$dy = y'(t)\,dt$

$$\bar{x} = \frac{1}{2A} \oint_C x^2\,dy$$

$$\bar{y} = -\frac{1}{2A} \oint y^2\,dx$$



① $x(t) = ta \qquad 0 \le t \le 1$

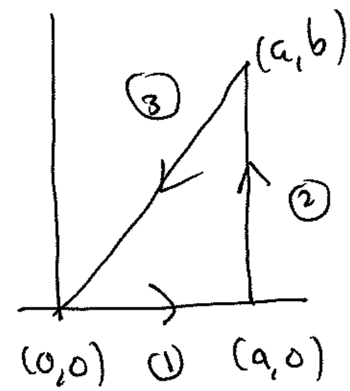
$y(t) = 0$

② $x(t) = a \qquad 0 \le t \le 1$

$y(t) = bt$

③ $x(t) = a(1-t) \qquad 0 \le t \le 1$

$y(t) = b(1-t) \qquad 0 \le t \le 1$



$A = \frac{1}{2}ab$

$$\bar{x} = \frac{1}{ab}\left[\oint_C x^2\,dy\right]$$

$$= \frac{1}{ab}\left[\cancel{0} + \int_0^1 a^2 b\,dt + \int_0^1 a^2(1-t)^2(-b)\,dt\right]$$

$$= \frac{1}{ab}\left[a^2 b + (-a^2 b)\int_0^1 (1-t)^2\,dt\right] \qquad u = 1-t$$

$$= \frac{1}{ab}\left[a^2 b + (-a^2 b)\int_0^1 u^2\,du\right]$$

$$= \frac{1}{ab}\left[a^2 b - a^2 b \cdot \left(\frac{1}{3}\right)\right] = \frac{1}{ab}\left[\frac{2}{3}a^2 b\right] = \frac{2a}{3}$$

$$= \frac{2}{8} a$$

<u>Theorem:</u> If $\vec{F} = P\hat{i} + Q\hat{j}$ and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in a simply connected region, then $\vec{F}$ is a conservative vector field.

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= 0$$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ is a measure of "how non-conservative" $\vec{F}$ is.