$$\text{comp}_{\vec{a}}\, \vec{b} = |\vec{b}| \cos\theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\, |\vec{b}| \cos\theta \qquad -(1)$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = |\vec{b}| \cos\theta \qquad -(2)$$

$$\text{proj}_{\vec{a}}\, \vec{b} = \underbrace{\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}\right)}_{\text{magnitude}} \cdot \underbrace{\frac{\vec{a}}{|\vec{a}|}}_{\text{direction}}$$

Find $\text{proj}_{\vec{a}}\, \vec{b}$ if $\vec{b} = \langle 4, 6 \rangle$

$\vec{a} = \langle -5, 12 \rangle$

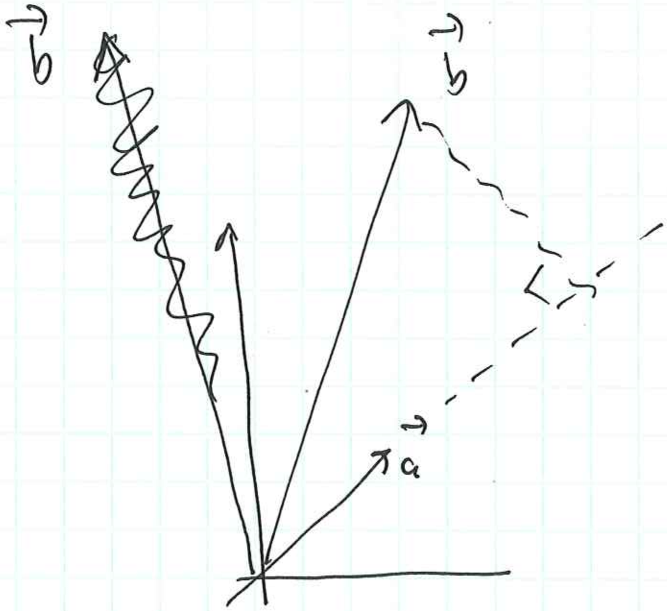
$$\vec{a} \cdot \vec{b} = (-5)(4) + (12)(6) = -20 + 72 = 52$$

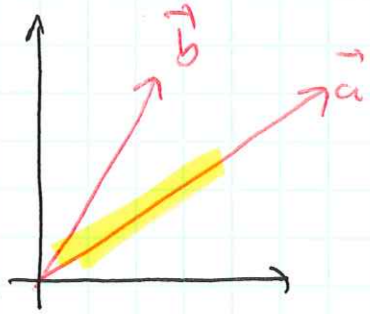
$$|\vec{a}|^2 = \vec{a} \cdot \vec{a} = (-5)^2 + (12)^2 = 25 + 144 = 169$$

$$\text{comp}_{\vec{a}}\, \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{52}{\sqrt{169}} = \frac{52}{13} = 4$$

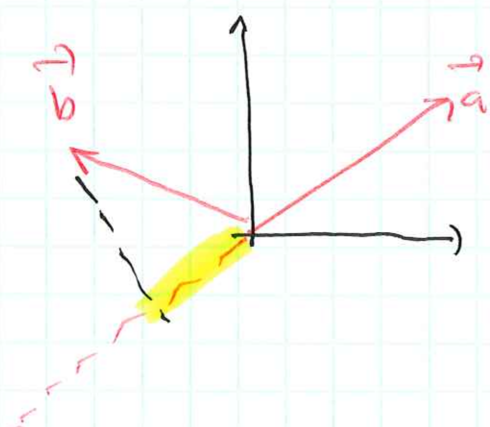
$$\text{proj}_{\vec{a}}\, \vec{b} = (4) \cdot \frac{\langle -5, 12 \rangle}{13}$$

$$= \frac{\langle -20, 48 \rangle}{13} = \left\langle -\frac{20}{13}, \frac{48}{13} \right\rangle$$

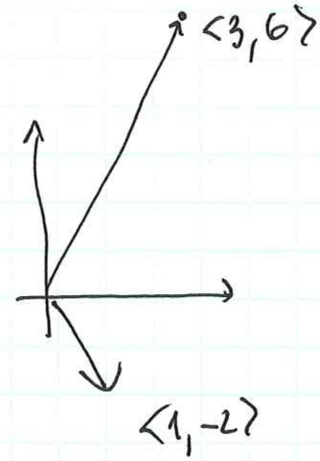




$\text{comp}_{\vec{a}}\,\vec{b}$ is <u>positive</u>



$\text{comp}_{\vec{a}}\,\vec{b}$ is <u>negative</u>



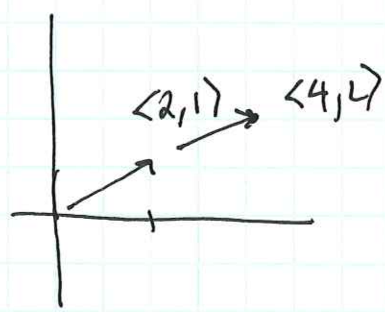
$$\begin{vmatrix} 1 & 3 \\ -2 & 6 \end{vmatrix} = 1 \cdot 6 - (-2) \cdot 3$$

$$= 1 \cdot 6 + 2 \cdot 3$$

$$= 12$$

$$\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - 4 \cdot 1$$

$$= 0$$



$$\begin{vmatrix} 2 & 1 \\ 4 & -6 \end{vmatrix} = 2 \cdot (-6) - 1 \cdot 4 = -12 - 4 = -16$$

$$\begin{vmatrix} 4 & -6 \\ 2 & 1 \end{vmatrix} = 4 \cdot 1 - (-6) \cdot 2 = 4 + 12 = 16$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}$$

$$+ a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -4 & -2 \\ 2 & 0 & 4 \\ -1 & 2 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 4 \\ 2 & 3 \end{vmatrix} - (-4) \begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix}$$

$$+ (-2) \begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix}$$

Most

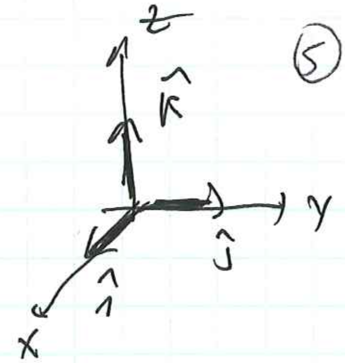
Maybe add challenge problem

$$= 1 \cdot (0 \cdot 3 - 4 \cdot 2) + 4(2 \cdot 3 - 4 \cdot (-1))$$

$$+ (-2)(2 \cdot 2 - 0 \cdot (-1))$$

$$= -8 + 4 \cdot (6 + 4) + (-2)(4)$$

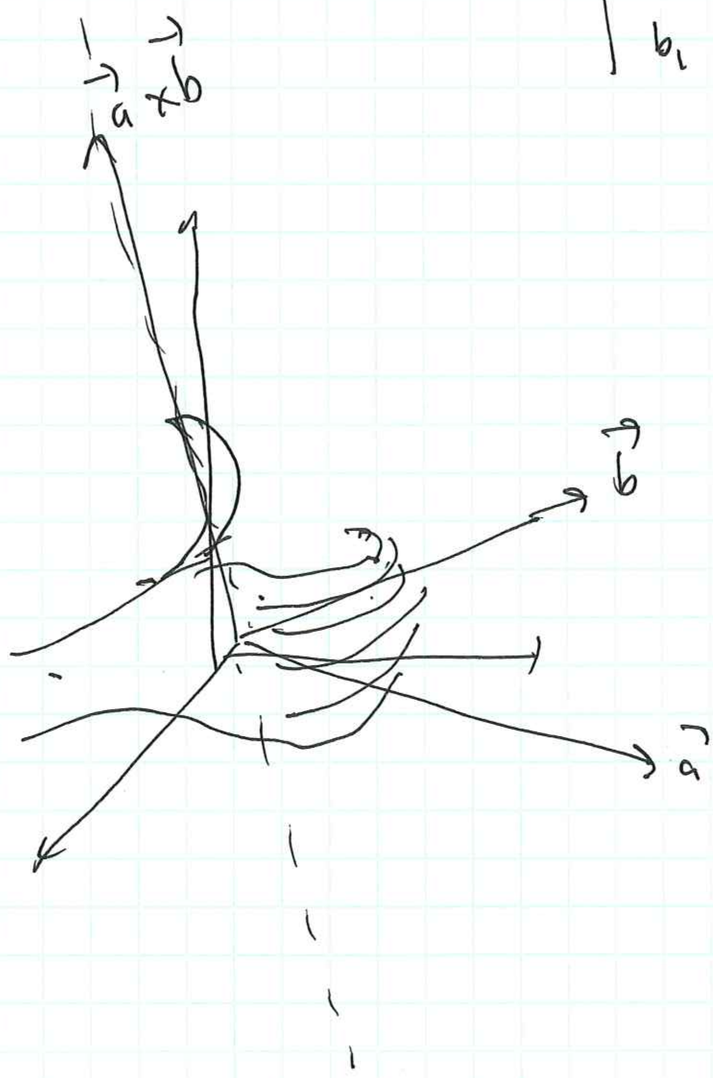
$$= -8 + 40 - 8 = 24$$



$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \hat{j}$$

$$+ \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \hat{k}$$



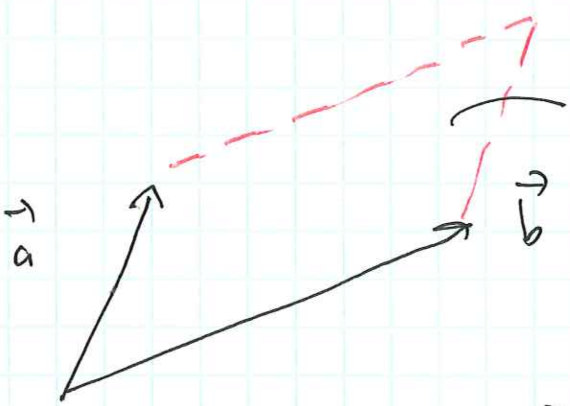
$\vec{a} = \langle 1, 0, 2 \rangle \qquad \vec{b} = \langle 1, 1, 1 \rangle$

$\vec{a} \times \vec{b} = \langle -2, 1, 1 \rangle$

$\vec{a} \cdot (\vec{a} \times \vec{b}) = 1 \cdot (-2) + 0 \cdot 1 + 2 \cdot 1 = 0$

$\vec{b} \cdot (\vec{a} \times \vec{b}) = 1(-2) + 1 \cdot 1 + 1 \cdot 1 = 0$

$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = \langle 2, -1, -1 \rangle$



Area $= |\vec{a} \times \vec{b}|$

$\vec{a} \perp \vec{b}$ if and only if $\vec{a} \cdot \vec{b} = 0$

$\vec{a} \parallel \vec{b}$ if and only if $\vec{a} \times \vec{b} = 0$