$$\vec{F} = P\,\hat{i} + Q\,\hat{j} + R\,\hat{j} \qquad P = P(x, y, z)$$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + \hat{j}\left(-\frac{\partial R}{\partial x} + \frac{\partial P}{\partial z}\right)$$

$$\hat{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$


$$\operatorname{curl} \vec{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} +$$


$$\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\hat{j} +$$

$$\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

$\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \hat{i}(0-0) + \hat{j}(0-0) + \hat{k}(0-0) = \vec{0}$$

$\vec{F} = yi - xj + 0\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix} =$$

$$(\ 0\ )\hat{i} + (\ 0\ )\hat{j}$$

$$+ \left( \underbrace{\frac{\partial}{\partial x}(-x)}_{-1} - \underbrace{\frac{\partial}{\partial y}(y)}_{-1} \right) \hat{k}$$

$$\nabla \times (\nabla f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

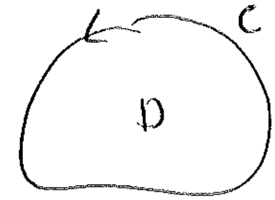
$$= \hat{i}\left( \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right)\right)$$

$$+ \cdots$$

$$= \hat{i}\left( \frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right) \rightarrow 0$$

$$+ \cdots$$

Green's Theorem




$$\iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA =$$

$$\oint_C P\,dx + Q\,dy$$

$\vec{F} = P\hat{i} + Q\hat{j}$ is gradient field if

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

on a <u>simply connected region</u>.

11/30/2018 5

$$\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$\text{div } \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

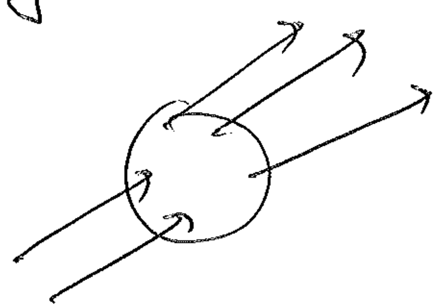
$$= \left\langle \underset{-}{\frac{\partial}{\partial x}}, \underset{+}{\frac{\partial}{\partial y}}, \underset{\#}{\frac{\partial}{\partial z}} \right\rangle \cdot \left\langle \underset{-}{P}, \underset{+}{Q}, \underset{\#}{R} \right\rangle$$

$$= \underset{-}{\frac{\partial P}{\partial x}} + \underset{+}{\frac{\partial Q}{\partial y}} + \underset{\#}{\frac{\partial R}{\partial z}}$$

$$\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 3$$

origin



Out far

$$\vec{F} = y\hat{i} - x\hat{j} + 0\hat{k}$$

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(-x) + \frac{\partial}{\partial z}(0)$$

$$= 0 + 0 + 0$$

---

$$\vec{\nabla} \times (\vec{\nabla} f) = 0 \quad \text{(Recall)}$$

---

$$\nabla \cdot (\vec{\nabla} \times \vec{F}) = 0$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$$

$$= \hat{i}\left( \frac{\partial}{\partial y}(3xy^2z^2) - \frac{\partial}{\partial z}(2xyz^3) \right)$$

$$6xyz^2 - 6xyz^2 = 0$$

$$+ \hat{j}\left( \frac{\partial}{\partial z}(y^2z^3) - \frac{\partial}{\partial x}(3xy^2z^2) \right)$$

$$3y^2z^2 - 3y^2z^2 = 0$$

$$+ \hat{k}\left( \frac{\partial}{\partial x}(2xyz^3) - \frac{\partial}{\partial y}(y^2z^3) \right)$$

$$2yz^3 - 2yz^3 = 0$$

$$= \vec{0}$$

Find $f$ so that

(1) $\dfrac{\partial f}{\partial x} = y^2 z^3$

(2) $\dfrac{\partial f}{\partial y} = 2xyz^3$

(3) $\dfrac{\partial f}{\partial z} = 3xy^2z^2$

---

(1) $f(x,y,z) = y^2z^3x + C(y,z)$

(2) $\cancel{2xyz^3 + \dfrac{\partial C}{\partial y}(y,z) = 2xyz^3}$

From (1), $\dfrac{\partial f}{\partial y} = \cancel{2yz^3x} + \dfrac{\partial C}{\partial y}(y,z) = \cancel{2xyz^3}$

$$\frac{\partial C}{\partial y}(y,z) = 0$$

$$C(y,z) = G(z)$$

$f(x,y,z) = xy^2z^3 + G(z)$

(3) $\cancel{3xy^2z^2} + G'(z) = \cancel{3xy^2z^2}$

$$G'(z) = 0$$

$$\boxed{f(x, y, t) = y^2 z^3 x + C}$$