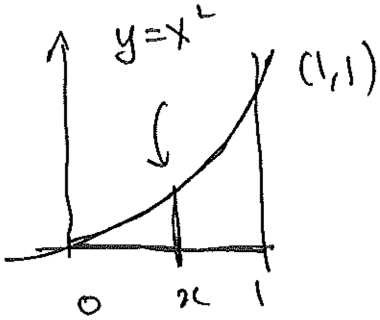
Type I and Type II

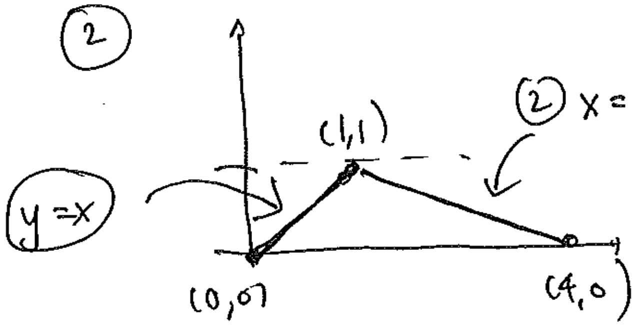
①



$$\int_0^1 \int_0^{x^2} x\cos(y)\,dy\,dx$$

$R: \{(x,y): 0 \le x \le 1, \quad 0 \le y \le x^2\}$

②



② slope: $\frac{-1}{3}$

$$y = -\frac{1}{3}x + C$$

$$0 = -\frac{1}{3}4 + C$$

$$C = \frac{4}{3}$$

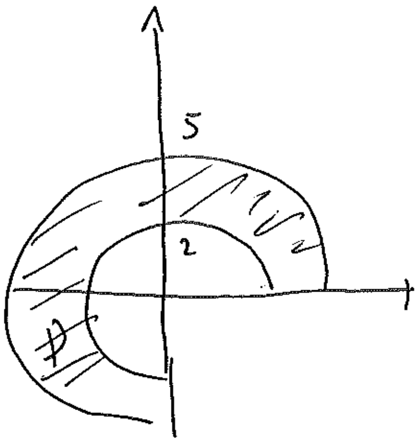
②: $y = -\frac{1}{3}x + \frac{4}{3}$

$$3y = -x + 4$$

$$x = 4 - 3y$$

$$\int_0^1 \left(\int_y^{4-3y} y \, dx \right) dy$$

$R: \{(x,y): 0 \le y \le 1, \quad y \le x \le 4-3y\}$



$$\{(r,\theta): \underline{2} \le r \le \underline{5},\ \underline{0} \le \theta \le \underline{\tfrac{3\pi}{2}}\}$$

$$\int_0^{3\pi/2}\int_2^5 \underline{f(r\cos\theta, r\sin\theta)\ r}\, dr\, d\theta$$

$$= \iint_D f(x,y)\, dA$$

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①: view as a solid over the yz plane:

$$D = \{(y,z): y^2+z^2 \le 9\}$$

$$E = \{(x,y,z): -2 \le x \le 2,\ y^2+z^2 \le 9\}$$

$$y = r\cos\theta$$

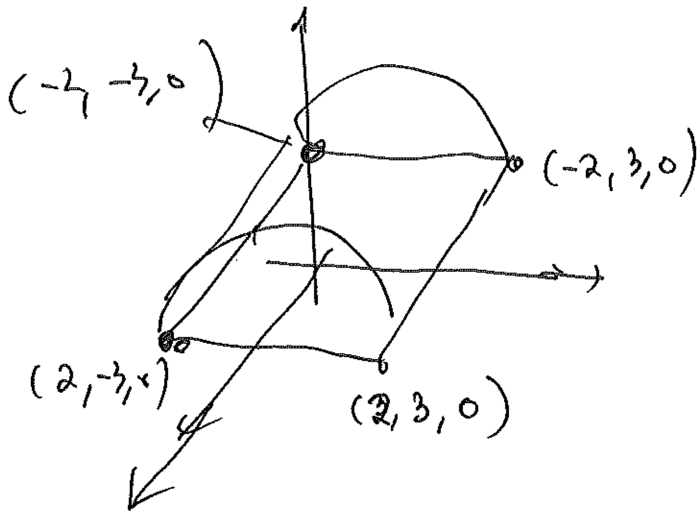
$$z = r\sin\theta$$

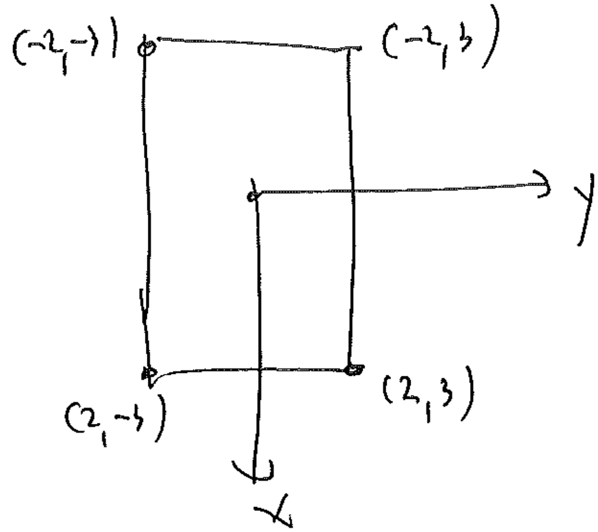
$$E = \{(x,r,\theta): -2 \le x \le 2,\ 0 \le r \le 3,\ 0 \le \theta \le 2\pi\}$$

$$\iiint_E f(x,y,z)\, dV = \int_0^{2\pi}\int_0^{3}\int_{-2}^{2} f(\qquad)\, dx\, r\, dr\, d\theta$$

$x,\ r\cos\theta,\ r\sin\theta$

A $\equiv$





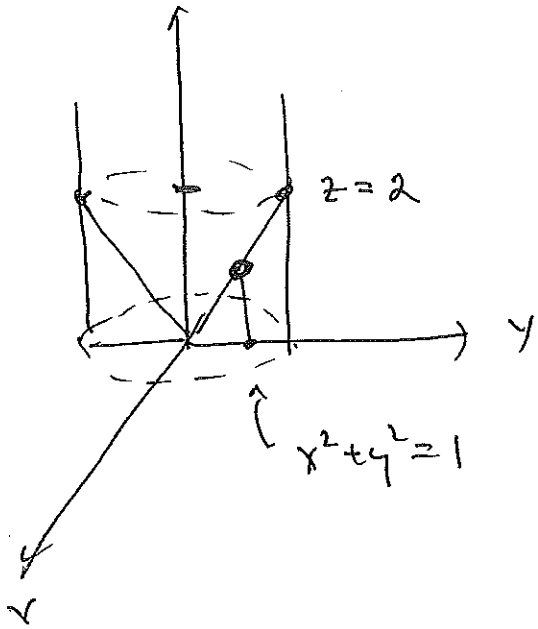
$$E = \{(x,y,z) : -2 \le x \le 2,\ -3 \le y \le 3,\ -\sqrt{9-y^2} \le z \le \sqrt{9-y^2}\}$$

$$y^2+z^2=9$$

$$z^2 = 9-y^2$$

$$z = \pm\sqrt{9-y^2}$$

$$\iiint_E f(x,y,z)\,dV = \int_{-2}^{2}\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y,z)\,dz\,dy\,dx$$



$\begin{cases} \text{Cone: } z^2 = 4(x^2+y^2) \\ \text{cyl: } x^2+y^2=1 \end{cases}$

Intersection: $z^2 = 4$

$z = \pm 2$

$\begin{cases} x^2+y^2 = 1 \\ z^2 = 4(x^2+y^2) \end{cases}$

substitute $x^2+y^2=1$

$z^2 = 4$

$z = \pm 2$

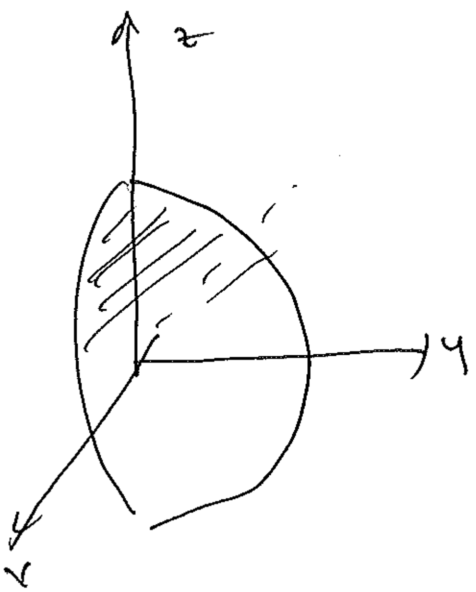
$E = \{ (x,y,z) : x^2+y^2 \le 1, \; 0 \le z \le 2\sqrt{x^2+y^2} \}$

$= \{ (r,\theta,z) : 0 \le r \le 1, \; 0 \le \theta \le 2\pi, \; 0 \le z \le 2r \}$

$$\iiint_E x^2 \, dV = \int_0^{2\pi} \int_0^1 \int_0^{2r} x^2 \; dz \; r\,dr \; d\theta$$

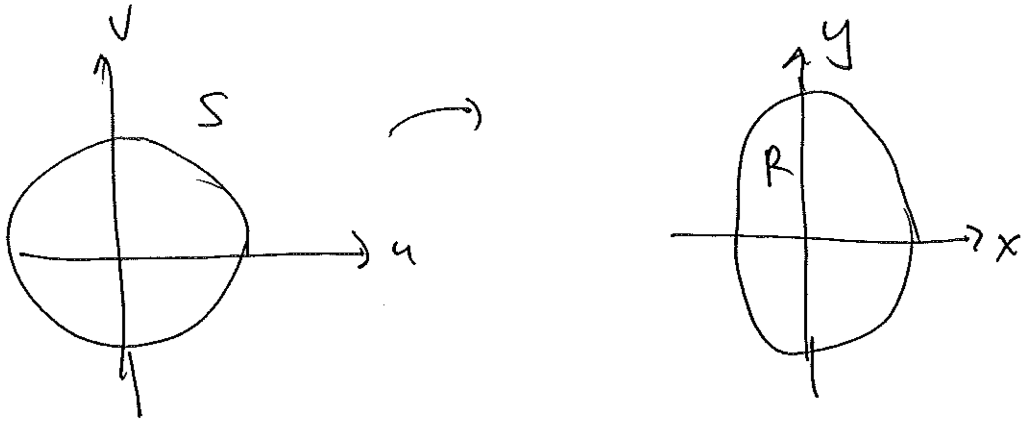
Spherical coordinates

$$x^2+y^2+z^2 \le 9, \quad y > 0$$



$$0 \le \theta \le \pi$$
$$0 \le \phi \le \pi$$
$$0 \le \rho \le 3$$

$$\iint_R f(x,y)\, dA = \iint_S f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\, dv$$

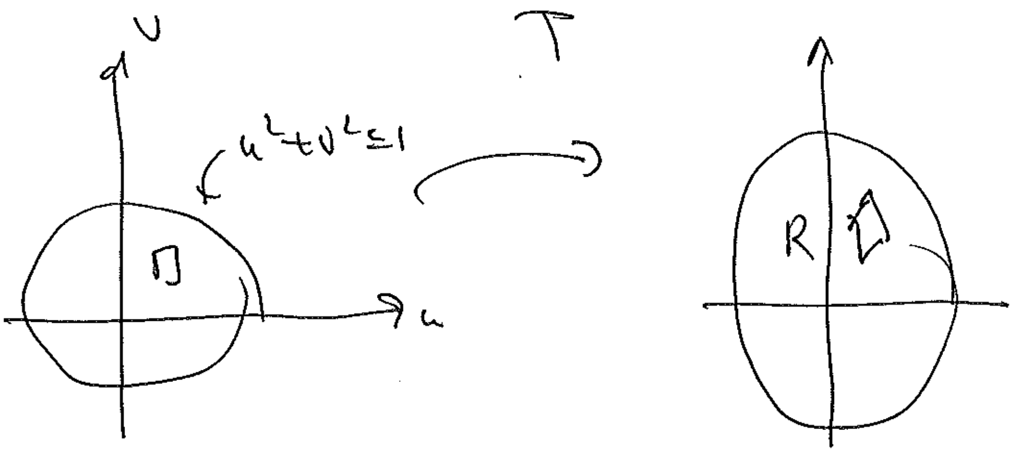


$$\boxed{x = 2u \quad y = 3v}$$

$$9x^2 + 4y^2 = 36$$

$$9 \cdot 4u^2 + 4 \cdot 9v^2 = 36$$

$$u^2 + v^2 = 1$$



$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$$

$$\iint_R x^2 \, dA = \iint_S (2u)^2 \cdot 6 \; du\, dv$$

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$u = r\cos\theta$ $\quad 0 \le r \le 1$

$v = r\sin\theta$ $\quad 0 \le \theta \le 2\pi$

$$\iint_S (2u)^2 \cdot 6 \; du\, dv =$$

$$\int_0^{2\pi} \int_0^1 (2r\cos\theta)^2 \, 6 \cdot r \, dr\, d\theta$$