Calculus III Meets the Final

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Homework

- Review for Final Exam on Thursday, December 13, 6:00-8:00 PM
- Be sure you know which room to go to for the final!

Final Exam Room Assignments

Sections 001-008	BE 111
Sections 009-013	**BS 107**
Sections 014-016	KAS 213

The Big Picture

Unit I	Vectors and Space Curves
Unit II	Differential Calculus
Unit III	Double and Triple Integrals
Unit IV	Calculus of Vector Fields

Goals of the Day

- Craft A Cheat Sheet
- Practice Problems
- Eat Cookies

Unit I: Moving Around in Space (1 of 2)

1. Dot product (scalar)

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\,\cos\theta = 0 \text{ if } \mathbf{a} \perp \mathbf{b}$$

   Geometric: Projection of one vector onto another

2. Cross product (vector)

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{0} \text{ if } \mathbf{a} \parallel \mathbf{b}$$

   Geometric: area of parallelogram spanned by **a** and **b**

3. Triple product (scalar)

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 \text{ if } \mathbf{a}, \mathbf{b}, \mathbf{c} \text{ coplanar}$$

   Geometric: Volume of parallelpiped spanned by **a**, **b**, and **c**

Unit I: Moving Around in Space (2 of 2)

4. Equation of line $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$
5. Equation of plane $ax + by + cz = d$, $\mathbf{n} = \langle a, b, c \rangle$
6. Quadric surfaces; cylinder, ellipsoid, paraboloid, saddle
7. Parametric curve $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$
8. Tangent vector $\mathbf{r}'(t)$
9. Projectile problems

Unit II: Differential Calculus (1 of 3)

1. Chain rule for partial derivatives
2. Gradient
$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$
points in direction of greatest change (increase)
3. Directional derivative $D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$
4. Critical points $\nabla f(a, b) = 0$

Unit II: Differential Calculus (2 of 3)

5. Second derivative test: Hessian matrix

$$H(f)(a,b) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}(a,b) & \frac{\partial^2 f}{\partial x \partial y}(a,b) \\ \frac{\partial^2 f}{\partial y \partial x}(a,b) & \frac{\partial^2 f}{\partial y^2}(a,b) \end{pmatrix},$$

Determinant

$$D(a,b) = \det H(a,b)$$

Second derivative test $D > 0$ (max or min), $D < 0$ (saddle)

If $D > 0$ and $f_{xx}(a,b) > 0$, local min

If $D > 0$ and $f_{xx}(a,b) < 0$, local max

6. Optimization of $f(x,y)$ over domain D: Find local extrema in D and ∂D

Unit II: Differential Calculus (3 of 3)

7. Lagrange Multipliers (Two variables, one constraint)
   Minimize $f(x, y)$ subject to $g(x, y) = 0$: solve

$$\nabla f(x, y) = \lambda \nabla g(x, y)$$
$$g(x, y) = 0$$

8. Lagrange mutlipliers (three variables, one constraint)
   Minimize $f(x, y, z)$ subject to $g(x, y, z) = 0$: solve

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$
$$g(x, y, z) = 0$$

Unit III: Integral Calculus

1. Double integrals $\iint_D f(x,y)\,dA$:

   Type I regions, Type II regions, polar coordinates (factor $r\,dr\,d\theta$)

2. Triple integrals $\iiint_E f(x,y,z)\,dV$:

   Type I, II and III regions
   Cylindrical coordinates (factor $r\,dr\,d\theta\,dz$)
   Spherical coordinates (factor $\rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$)

3. Change of variables theorem: If $T : S \to D$ $(x = x(u,v),\ y = y(u,v))$ then

$$\iint_D f(x,y)\,dA = \iint_S f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dudv$$

Unit IV: Vector Calculus (1 of 2)

1. Line integral of a scalar function:

$$\int_C f(x,y)\,ds, \quad \int_C f(x,y)\,dx, \quad \int_C f(x,y)\,dy$$

   (parameterize!)

2. Line integral of a vector function:

$$\int_C \mathbf{F}(x,y)\cdot\,d\mathbf{r}, \quad \int_C \mathbf{F}(x,y,z)\cdot\,d\mathbf{r}$$

   (ditto!)

3. Green's Theorem:

$$\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA = \oint_C P\,dx + Q\,dy$$

Unit IV: Vector Calculus (2 of 2)

5. Curl (vector) - measures 'rotation'. If

$$\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k},$$

then

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

6. Divergence (scalar) - measures 'outflow.' If

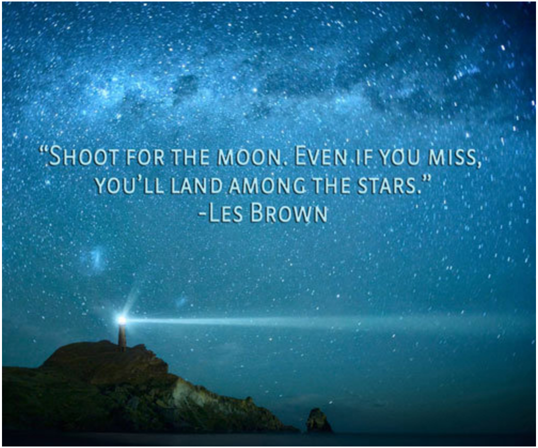
$$\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k},$$

then

$$\nabla \cdot F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

More Review Problems

1. Find the equation of a line through the point $(6, -5, 2)$ and parallel to the vector $\langle 1, 3, -2/3 \rangle$
2. Find the equation of the plane that contains the points $(3, -1, 1)$, $(4, 0, 2)$, and $(6, 3, 1)$
3. An athlete throws a shot at an angle of 45^o to the horizontal at an initial speed of 43ft/sec. It leaves her hand 7ft above the ground. Where does the shot land?
4. If $v = x^2 \sin y + ye^{xy}$, $x = s + 2t$, $y = st$, use the chain rule to find $\partial v/\partial s$ and $\partial v/\partial t$ when $s = 0$ and $t = 1$.
5. Use Lagrange multipliers to find the maximum of $f(x, y, z) = xyz$ if $x^2 + y^2 + z^2 = 3$.


"SHOOT FOR THE MOON. EVEN IF YOU MISS,
YOU'LL LAND AMONG THE STARS."
-LES BROWN

Try to be good, but always be kind.

—The Twelfth Doctor