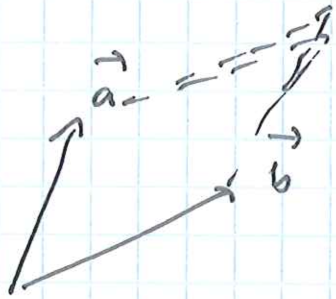
Cross Product / Lines and Planes 8/31/2018



$|\vec{a} \times \vec{b}|$ = Area of parallelogram spanned by $\vec{a}$, $\vec{b}$

$$\vec{PQ} = \langle 2, 3, 1 \rangle$$

$$\vec{PS} = \langle 4, 2, 5 \rangle$$

$$\vec{PQ} \times \vec{PS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 4 & 2 & 5 \end{vmatrix}$$

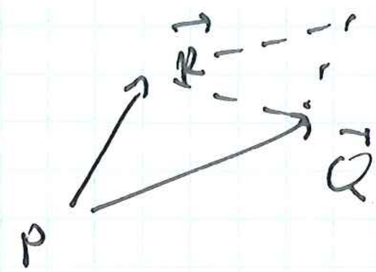
$$= \hat{i} \begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix}$$

$$= 13\hat{i} - 6\hat{j} + -8\hat{k}$$

$$|\vec{PQ} \times \vec{PS}| = \sqrt{13^2 + 6^2 + 8^2}$$

$\vec{PR} = \langle 3, 2, 4 \rangle$

$\vec{PQ} = \langle -3, 1, 2 \rangle$



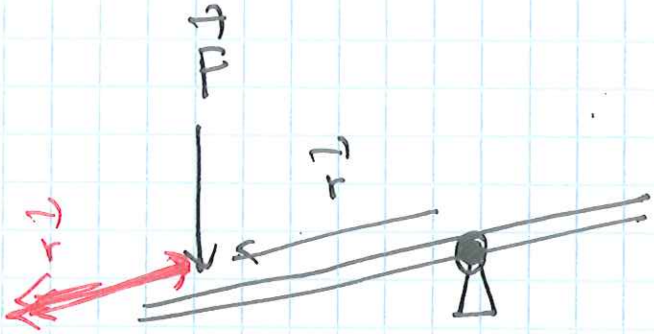
$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

For parallelepiped:

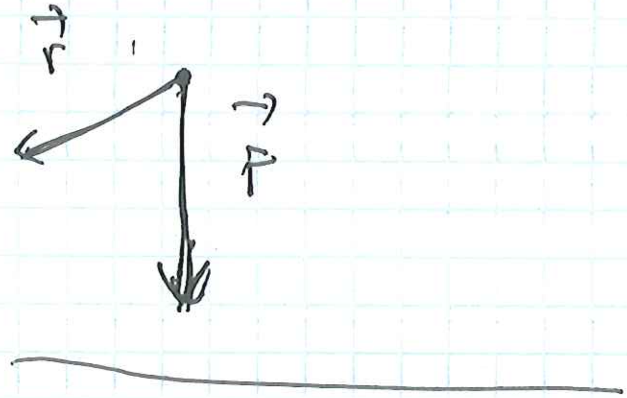
$$\vec{PQ} \cdot (\vec{PR} \times \vec{PS}) = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 3 & -1 \\ 5 & 5 & 1 \end{vmatrix}$$

$$= 4 \begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} - 2 \begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} + 2 \begin{vmatrix} 3 & 3 \\ 5 & 5 \end{vmatrix}$$

Torque



$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$P_0 = (x_0, y_0, z_0)$$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\vec{v} = \langle a, b, c \rangle$$

$$\vec{r}(t) = \cancel{\langle a, b, c} \ \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

$$\langle x(t), y(t), z(t) \rangle = \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$$

1) $(x_0, y_0, z_0) = (1, 2, -1)$

$\vec{v} = \overrightarrow{PQ} = \langle 1, 1, 5 \rangle$

$x(t) = 1 + 1t$

$y(t) = 2 + 1t$

$z(t) = -1 + 5t$

so e.g. at $t = 1$

$x(1) = \cancel{3}\ 2$

$y(1) = 3$

$z(1) = 4$

$t = 2$: $x(t) = 3$

$y(t) = 4$

$z(t) = 9$

2) Point on the line $(1, 2, 3)$

Vector along the line: $\langle 2, -3, 4 \rangle$

$x(t) = 1 + 2t$

$y(t) = 2 + -3t$

$z(t) = 3 + 4t$

$\cancel{x(t) = x_0 + b}$

$x(t) = x_0 + at$

$y(t) = y_0 + bt$

$z(t) = z_0 + ct$

$x - x_0 = at$

$y - y_0 = bt$

$z - z_0 = ct$

$\frac{x - x_0}{a} = t$

$\frac{y - y_0}{b} = t$

$\frac{z - z_0}{c} = t$

1) Point on the line: $(0,0,0)$

Vector along the line: $\langle \underset{a}{4}, \underset{b}{3}, \underset{c}{-1} \rangle$

Parametric Eq'ns:

$$x(t) = 0 + 4t$$
$$y(t) = 0 + 3t$$
$$z(t) = 0 + -1t$$

Symmetric Eq'n's:

$$\frac{x-0}{4} = \frac{y-0}{3} = \frac{z-0}{(-1)}$$

$$\frac{x}{4} = \frac{y}{3} = -(z)$$

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2) Point: $(2,1,0)$

Vector: $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}\hat{k}$

$= \hat{i} - \hat{j} + \vec{k}$

$= \langle 1, -1, 1 \rangle$

$\quad\; x_0\; y_0\; z_0$

Point $(2, 1, 0)$

vector $\langle 1, -1, 1 \rangle$

$\qquad\quad\; a \;\; b \;\; c$

Parametric:

$$x(t) = 2 + 1t$$
$$y(t) = 1 + -1t$$
$$z(t) = 0 + 1t$$

symmetric:

$$\frac{x - \overset{x_0}{2}}{\underset{a}{1}} = \frac{y - \overset{y_0}{1}}{\underset{b}{-1}} = \frac{z - \overset{z_0}{0}}{\underset{c}{1}}$$

Plane thru $(0,0,0) = (x_0, y_0, z_0)$

$\vec{n} = \langle -1, 2, 5 \rangle$

(a, b, c)

$-1(x) + 2(y-0) + 5(z-0) = 0$

$-x + 2y + 5z = 0$