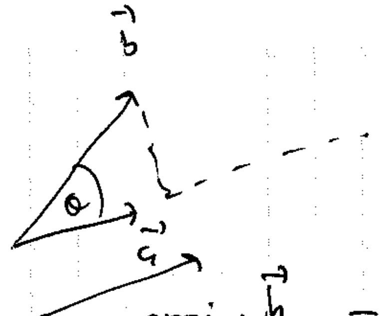
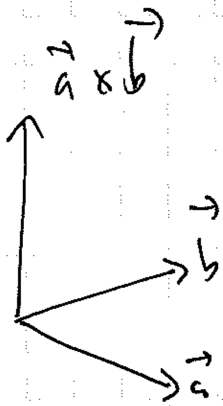
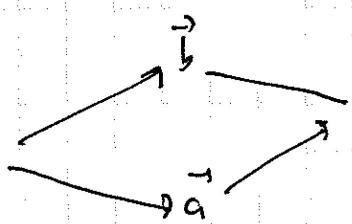
Dot



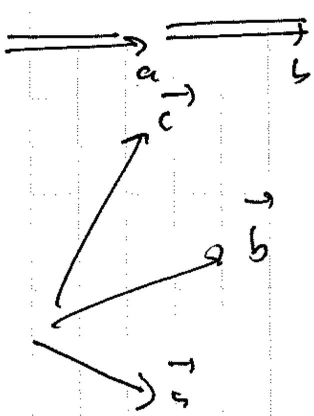
$$\mathrm{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$$

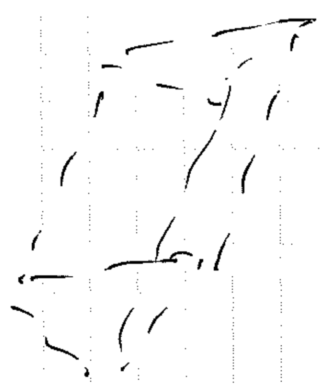
Cross



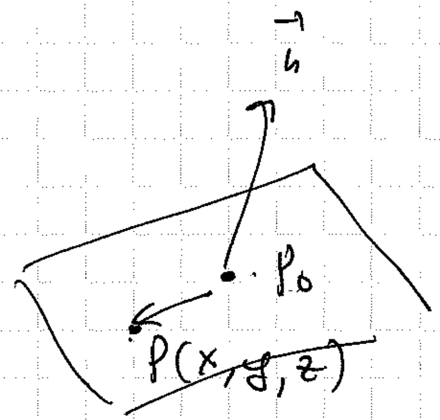


Area $\Diamond = |\vec{a} \times \vec{b}|$





Eq'n of plane



$P_0(x_0, y_0, z_0)$

$\vec n = \langle a, b, c \rangle$

$\overrightarrow{P_0P} \cdot \vec n = 0$

$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$ax + by + cz = ax_0 + by_0 + cz_0 = d$

$\underset{\uparrow}{a}x + \underset{\uparrow}{b}y + \underset{\uparrow}{c}z = d$

determined by $\vec n$

$P_0 = P_0(2, 2, 2)$

$\vec n = \langle 1, -1, 2 \rangle$

Eq'n of plane $\quad x - y + 2z = \square$

To find $\square$, plug in $(x, y, z) = (2, 2, 2)$

$$1\cdot 2 - 1\cdot 2 + 2\cdot 2 = 4$$

$$\boxed{x - y + 2z = 4}$$

Plane: Orthogonal to $\langle x, y, z\rangle = (-7, 0, 0) + t(-7, 3, 3)$

Passes thru $(0, 0, -7)$

$$P_0 = (0, 0, -7)$$

$$\underset{}{\vec{n}} = \langle \underset{a}{-7}, \underset{b}{3}, \underset{c}{3} \rangle$$

$$-7x + 3y + 3z = \boxed{-21}$$

Switch: $7x - 3y - 3z = \boxed{}$

$$\cancel{7(0) + 3(0)} - 3$$

$$7(0) - 3(0) - 3(-7) = \boxed{21}$$

$$7x - 3y - 3z = 21$$

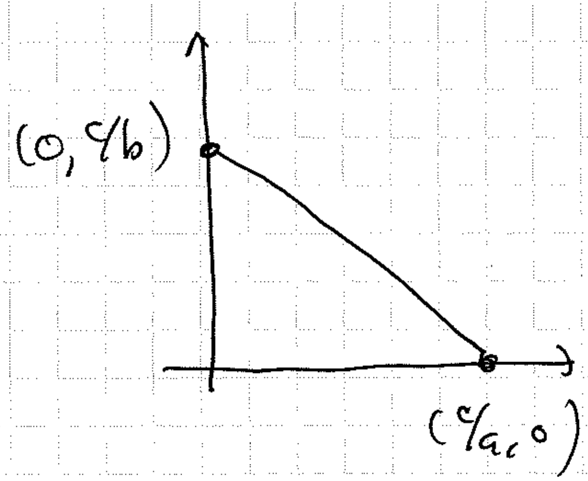
In 2-d, to graph

$$ax + by = c$$

$x = 0$

$by = c$

$y = c/b$

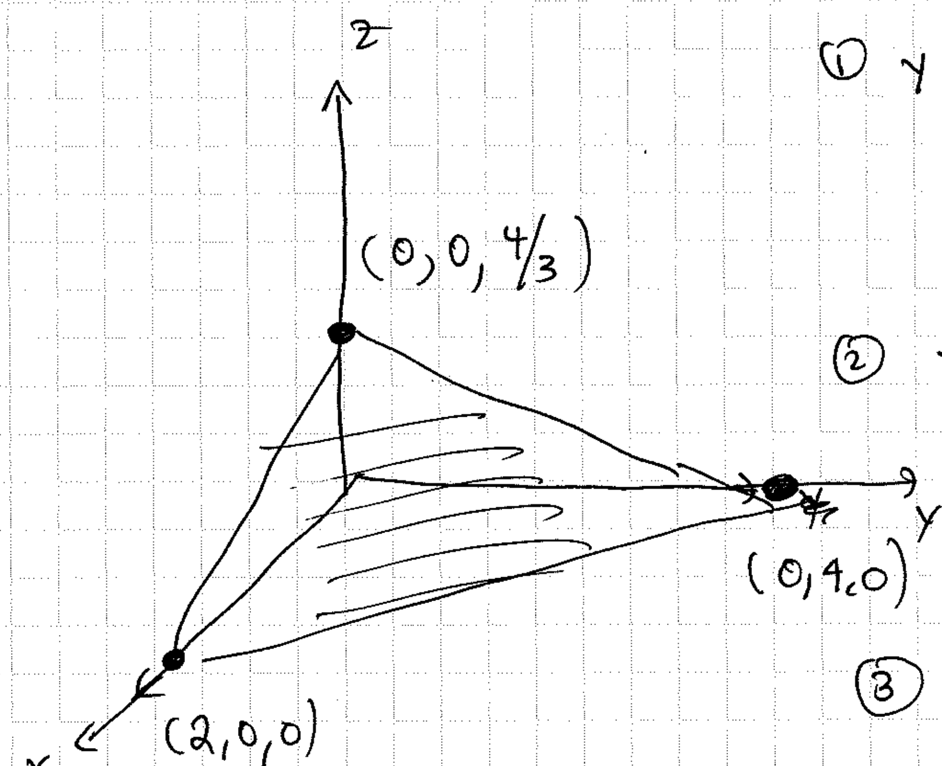


$y = 0$

$ax = c$

$x = c/a$

$$2x + y + 3z = 4$$



① $y = z = 0$

$2x = 4$

$x = 2$

② $x = z = 0$

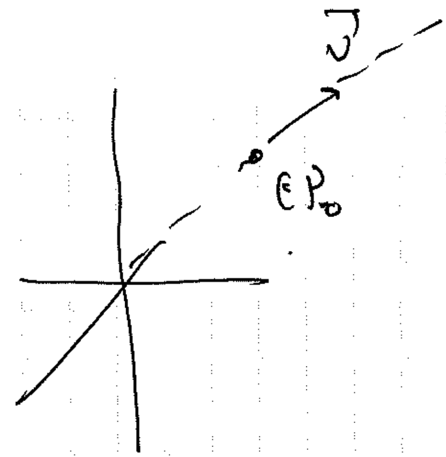
$y = 4$

③ $x = y = 0$

$3z = 4 \quad z = 4/3$

$P_0 : (x_0, y_0, z_0)$

$\vec{v} = \langle a, b, c \rangle$



$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\vec{v} = \langle a, b, c \rangle$$

Parametric

$$x = x_0 + at$$

$$y = y_0 + bt$$

$$z = z_0 + ct$$

Symmetric

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Ex:

L_1

$$x = 1 + 2t$$

$$y = 3 + 4t$$

$$z = 6t$$

$$\vec{v} \cdot \cancel{d} = \langle 2, 4, 6 \rangle$$

L_2

$$x = t$$

$$y = 2t$$

$$z = 3t$$

$$\vec{v} = \langle 1, 2, 3 \rangle$$

#1 To test for intersection

1) $x|$ $\quad 2+s = 3+t$

2) $y|$ $\quad 3-2s = -4+3t$

3) $z|$ $\quad 1-3s = 2-7t$

$$1) \quad 2+s = 3+t$$
$$2) \quad 3-2s = -4+3t$$

$\Rightarrow$

$$4+2s = 6+2t$$
$$\underline{3-2s = -4+3t}$$
$$7 = 2+5t$$
$$5 = 5t$$
$$t = 1$$

Now use $t=1$ in (1)

$$2+s = 4$$
$$s = 2$$

check 3) $\quad 1-3(2) = 2-7(1)$

$$-5 = -5$$

So intersection occurs at $s=2,\ t=1$

i.e. $(x,y,z) = (4,-1,-5)$

$\vec{n}_1 = \langle 1, 2, 3 \rangle$

$\vec{n}_2 = \langle 1, -1, 1 \rangle$

vector pointing along line of intersection:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 5\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\boxed{\vec{v} = \langle 5, 2, -3 \rangle}$$

$$\begin{array}{rl} & x + 2y + 3z = 1 \\ (-) & x - y + z = 1 \\ \hline & 3y + 2z = 0 \end{array}$$

$y = z = 0 \qquad x = 1$

$$\boxed{P_0 = (1, 0, 0)}$$