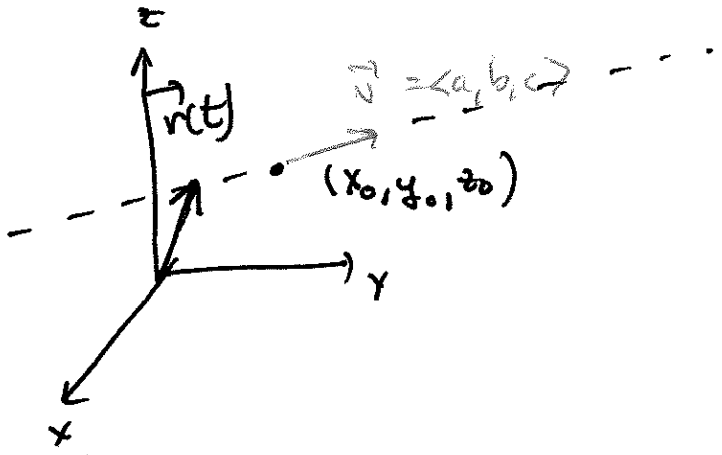
Vector Functions and Space Curves



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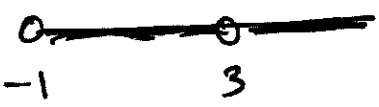
1) Domain:

$t+1 > 0$, $t \neq 3, -3$,

OR: $t > -1$, $t \neq 3$



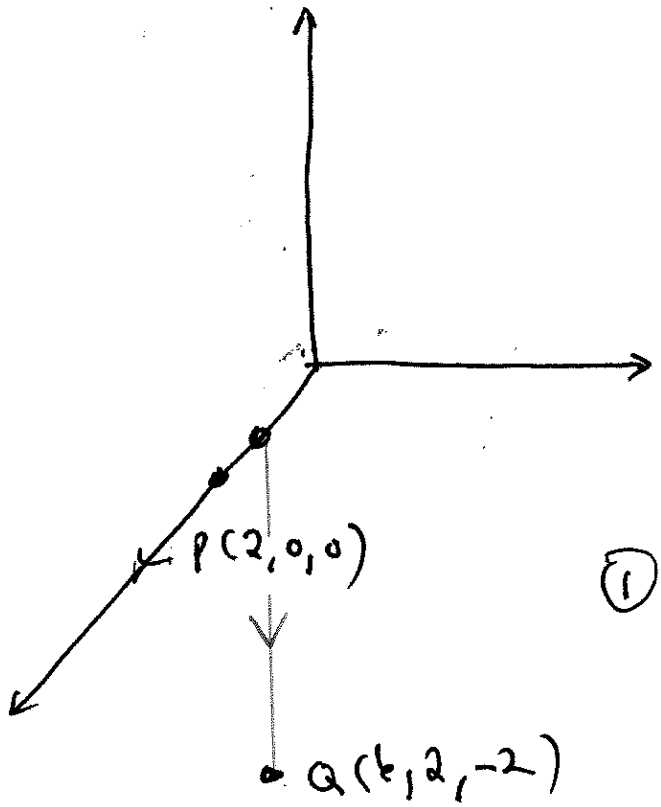
2) Limits:

$$\lim_{t \to 1} \left(\frac{t^2 - t}{t - 1} \right) = \lim_{t \to 1} \frac{t(t-1)}{(t-1)} = 1$$

$$\lim_{t \to 1} \sqrt{t+8} = 3$$

$$\lim_{t \to 1} \left(\frac{\sin \pi t}{\ln t} \right) = \lim_{t \to 1} \left(\frac{\pi \cos \pi t}{\frac{1}{t}} \right) = \frac{\pi(-1)}{1} = -\pi$$

ANS: $\langle 1, 3, -\pi \rangle$



$P(2,0,0)$

$\vec{v} = \vec{PQ}$

$= \langle 4, 2, -2 \rangle$

① $\vec{r}(t) = \langle 2,0,0 \rangle + t\langle 4,2,-2 \rangle$

↑ vector function

$t=0$ P

$t=1$ Q

② $0 \le t \le 1$

$x(t) = 2 + 4t$

$y(t) = 0 + 2t$

$z(t) = 0 - 2t$

} Parametric eq'n's

Projections:

1) xz plane $y = 0$

Projection: $\langle \cos t, 0, \sin t \rangle$
$\langle x, 0, z \rangle$

2) xy plane $z = 0$

Projection: $\langle \cos t, t, 0 \rangle$

3) yz plane $x = 0$

Projection: $\langle 0, t, \sin t \rangle$

Helix:

$$x(t) = \cos(t) \qquad y(t) = t \qquad z(t) = \sin(t)$$

$$x(t)^2 + z(t)^2 = \cos^2(t) + \sin^2(t) = 1$$

Spiral:

$$x(t) = t\cos t \qquad y(t) = t\sin t \qquad z(t) = t$$

$$x(t)^2 + y(t)^2 = t^2\cos^2 t + t^2\sin^2 t$$
$$= t^2$$

$$z(t)^2 = t^2$$

So: $x(t)^2 + y(t)^2 = z(t)^2$

$x(t) = \sin(t) \quad y(t) = \cos(t) \quad z(t) = \sin^2(t)$

• Lieson: 1) $z = x^2$

$x(t)^2 = \sin^2(t)$

$z(t) = \sin^2(t)$ ✓

2) $x^2 + y^2 = 1$

$x(t)^2 + y(t)^2 = \sin^2(t) + \cos^2(t) = 1$ ✓

Intersectia of Helix and Sphere

Find all t so that

$$\sin^2(t) + \cos^2(t) + t^2 = 5$$
$$1 + t^2 = 5$$
$$t^2 = 4$$
$$t = \pm 2$$

$\vec{r}(-2) = \langle \sin(-2), \cos(-2), -2 \rangle$

$\vec{r}(+2) = \langle \sin(2), \cos(2), +2 \rangle$

9/10/18 (6)

Collision:

$\vec{r}_1(t) = \vec{r}_2(t)$

1) $t = 1 + 2t \quad \Rightarrow \quad t = -1$

2) $t^2 = 1 + 6t$ Try $t=-1$: $1 = 1 + 6(-1) \neq 1$

3) $t^3 = 1 + 14t$

No collision

Intersection:

$\boxed{s = 1 + 2t}$ → Try

$s^2 = 1 + 6t$ 2) $(1+2t)^2 = 1 + 6t$

$s^3 = 1 + 14t$ 3) $(1+2t)^3 = 1 + 14t$

2) $1 + 4t + 4t^2 = 1 + 6t$

$4t^2 - 2t = 0$

$2t(2t - 1) = 0$

$t = 0, \; t = 1/2$

3) $t = 0$ works!