Quiz 10

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (3 points) Let C be the line segment from $(1, 0)$ to $(4, 4)$.

   (a) Consider the function, $f(x, y) = x + y$. Compute the line integral,

$$\int_C f(x, y)\,\mathrm{d}s$$

> **Solution:** We can parameterize C by
>
> $$\mathbf{r}(t) = \langle 4, 4\rangle t + \langle 1, 0\rangle(1 - t) = \langle 1 + 3t, 4t\rangle,$$
>
> $0 \leq t \leq 1$. Then $\mathbf{r}'(t) = \langle 3, 4\rangle$ and $|\mathbf{r}'(t)| = 5$, both constant in t. Then,
>
> $$\int_C (x + y)\,\mathrm{d}s = \int_0^1 ((1 + 3t) + (4t))5\,\mathrm{d}t = \frac{45}{2}$$

   (b) Consider the vector field, $\mathbf{G}(x, y) = \langle 2x, y\rangle$. Here, the curve C is the same as above. Compute the line integral,

$$\int_C \mathbf{G}(x, y) \cdot \mathrm{d}\mathbf{r}$$

> **Solution:** Using the parameterization from part (a), taking the line integral of the vector field, $\mathbf{G}$, we get,
>
> $$\int_C \mathbf{G}(x, y) \cdot \mathrm{d}\mathbf{r} = \int_0^1 \langle 2(1 + 3t), 4t\rangle \cdot \langle 3, 4\rangle\,\mathrm{d}t = \int_0^1 (6 + 34t)\,\mathrm{d}t = 23$$

2. (2 points) Let C be the curve given by $\mathbf{r}(t) = \langle t^2 + 1, 3^t \rangle$ for $0 \leq t \leq 1$. Consider the vector field, $\mathbf{F}(x, y) = \langle y, x \rangle$. Show that $\mathbf{F}(x, y)$ is a conservative vector field then compute the line integral,

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r}$$

Solution: Note that the function $f(x, y) = xy$ has the properties that $f_x(x, y) = y$ and $f_y(x, y) = x$ so that $\nabla f(x, y) = \mathbf{F}(x, y)$. Therefore, $\mathbf{F}$ is a conservative vector field. We can then use the fundamental theorem of calculus for line integrals to evaluate the desired integral. We note that the endpoints of the curve C are $\mathbf{r}(0) = \langle 1, 1 \rangle$ and $\mathbf{r}(1) = \langle 2, 3 \rangle$

So,

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r} = f(\mathbf{r}(1)) - f(\mathbf{r}(0)) = 6 - 1 = 5$$