Quiz 8

Name: _____   Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (3 points) Consider the lamina D which is defined as the region of the xy-plane

$$x \geq 0, \quad y \geq 0, \quad x^2 + y^2 \leq 4,$$

with density at a point (x,y) equal to the distance from (x,y) to the origin. **Set up** and **compute** an interated integral in polar coordinates which calculates the mass of this lamina.

> **Solution:** The region D is a polar rectangle with $0 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$. The density function in polar coordinates is given by $\rho(r,\theta) = r$. Hence the mass is given by the double integral
>
> $$\iint_D \rho(x,y)\, dA = \int_0^{\pi/2} \int_0^2 r \cdot r\, dr\, d\theta = \int_0^{\pi/2} \frac{1}{3}(2^3)\, d\theta = \frac{4\pi}{3}.$$

2. (2 points) **Set up** an interated integral which computes the surface area of the function $f(x,y) = 8 - 4x - 2y$ on the region bounded by the lines $y = 4 - 2x$, $x = 0$, and $y = 0$. Do not simplify.

> **Solution:** We compute $f_x(x,y) = -4$, $f_y(x,y) = -2$, so the surface area is given by
>
> $$\iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA = \int_0^2 \int_0^{4-2x} \sqrt{16 + 16 + 1}\, dy\, dx.$$