Math 213 - The Chain Rule and Implicit Differentiation

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Reminders

- Homework B3 on 14.4 is due Wednesday night
- Quiz # 4 on 14.1, 14.3 takes place on Thursday
- Homework B4 on 14.5 is due Friday night

Review O	f(x(t), y(t))	f(x(s,t),y(s,t))	The Chain Rule Tree 000	Implicit Differentiation 00
Unit II: I	Functions of	of Several V	ariables	
13.3-4 Leo	cture 11: Veloci	ty and Accelerati	on	

- 14.1 Lecture 12: Functions of Several Variables
- 14.3 Lecture 13: Partial Derivatives
- 14.4 Lecture 14: Linear Approximation
- 14.5 Lecture 15: Chain Rule, Implicit Differentiation
- 14.6 Lecture 16: Directional Derivatives and the Gradient
- 14.7 Lecture 17: Maximum and Minimum Values, I
- 14.7 Lecture 18: Maximum and Minimum Values, II
- 14.8 Lecture 19: Lagrange Multipliers
- 15.1 Double Integrals
- 15.2 Double Integrals over General Regions Exam II Review

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Learning Goals

- Review the chain rule for functions of one variable
- Learn how to differentiate f(x, y) along a curve (x(t), y(t))
- Learn how to differentiate f(x, y) along x(s, t), y(s, t)
- Learn about the Chain Rule Tree
- Learn about Implicit Differentiation

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Chain Rule for Functions of One Variable

The Chain Rule, 1 Variable If y = f(u) and u = u(x), then

 $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Remember that, at the end of the computation, you substitute for u the formula for u in terms of x!

- 1 If $y = u^3$ and $u = \cos x$, find dy/dx
- 2 Find the derivative of $g(x) = (x^2 + 1)^{3/2}$

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f(x(t), y(t))		
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Case 1: f(x(t), y(t))

The Chain Rule, 2 Variables (Case 1) If z = f(x, y), x = g(t), and y = h(t), then $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

1 Suppose that $z = \sin x \cos y$, $x = \sqrt{t}$, and y = 1/t. Find dz/dt.

2 Suppose that $z = \sqrt{1 + xy}$, $x = \tan t$, and $y = \arctan t$. Find dz/dt.

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f(x(t), y(t))		

Case 1: f(x(t), y(t))

The Chain Rule, 2 Variables (Case 1) If z = f(x, y), x = g(t), and y = h(t), then $\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$

The differential of z is

$$dz = \frac{\partial f}{\partial x} \, dx + \frac{\partial z}{\partial y} \, dy$$

If x = x(t) and y = y(t) then

$$dx = \frac{dx}{dt} dt, \quad dy = \frac{dy}{dt} dt$$

Hence

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

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Case 2: f(x(s,t), y(s,t))

The Chain Rule, 2 Variables (Case 2) If

$$z = f(x, y), x = g(s, t), y = h(s, t),$$

then

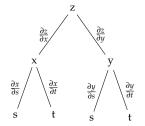
$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}, \qquad \qquad \frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t},$$

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The Chain Rule Tree



You can visualize the chain rule by a tree diagram: If

z = f(x, y)

and

$$x = g(s, t), \quad y = h(s, t),$$

then:

- You can find ∂z/∂s by adding contributions for all paths from z to s
- You can find ∂z/∂t by adding contributinos for all paths from z to t

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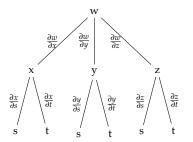
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The Chain Rule Tree

Use the following diagram to find formulas for $\partial w/\partial s$ and $\partial w/\partial t$ if w is a function of x, y, and z, and x, y, z are each functions of s and t



	The Chain Rule Tree	
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More Fun with the Chain Rule

- 1 Find $\frac{\partial z}{\partial t}$ if $w = \ln \sqrt{x^2 + y^2 + z^2}$, $x = \sin t$, $y = \cos t$, and $z = \tan t$
- 2 Find $\partial w / \partial r$ if w = xy + yz + xz, $x = r \cos \theta$, $y = r \sin \theta$, $z = r\theta$.
- **3** Suppose $g(u, v) = f(e^u + \sin v, e^u + \cos v)$. Use the following table to find $g_u(0, 0)$ and $g_v(0, 0)$.

	f	g	f_x	fy
(0,0)	3	6	4	8
(1,2)	6	3	2	5

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Implicit Differentiation

If *y* is defined implicitly as a function of *x* by the equation F(x, y) = 0, we can use the differential

$$dF = \frac{\partial F}{\partial x} \, dx + \frac{\partial F}{\partial y} \, dy$$

to find dy/dx.

If F(x, y) is constant, then

$$dF = 0 = \frac{\partial F}{\partial x} \, dx + \frac{\partial F}{\partial y} \, dy$$

and we can solve for dy/dx

We can use a similar technique for *z* defined implicitly as a function of *x*, *y* by an equation of the form G(x, y, z) = 0.

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Summary

- We reviewed the chain rule for functions of one variable
- We studied several cases of the chain rule for functions of several variables, beginning with z = f(x, y), x = g(t), y = h(t)
- We learned how to use the "chain rule tree" to apply the chain rule
- We learned how to compute partial derivatives of implicitly defined functions

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