Math 213 - Triple Integrals (Part I)

Peter A. Perry

University of Kentucky

October 23, 2019

Reminders

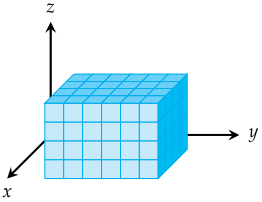
- Quiz # 6 on 15.1-15.2 is tomorrow in recitation. Study up!
- Homework C1 on section 15.3 is due Friday night

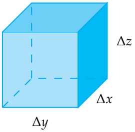
Unit III: Multiple Integrals, Vector Fields

Goals of the Day

- Understand triple integrals as a limit of Riemann sums
- Understand how to compute triple integrals as iterated integrals
- Understand how to compute triple integrals over fiendishly contrived regions

Riemann Sums





Given a rectangular box

$$B = [a, b] \times [c, d] \times [r, s]$$

and a function $f(x, y, z)$, we can divide the box into cubes of side Δx, Δy, Δz and volume

$$\Delta V = \Delta x \, \Delta y \, \Delta z$$

The *triple integral* of f over the box B is the limit of Riemann sums

$$\sum_{i,j,k} f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk}) \Delta V$$

and is denoted

$$\iiint_B f(x, y, z) \, dV$$

Triple Integrals as Iterated Integrals

If $B = [a,b] \times [c,d] \times [r,s]$ then

$$\iiint_B f(x,y,z)\,dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz$$

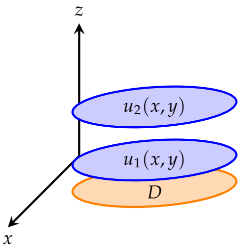
Evaluate $\iiint_B (xy + z^2)\,dV$ if

$$B = \{(x,y,z) : 0 \leq x \leq 2,\ 0 \leq y \leq 1,\ 0 \leq z \leq 3\}$$

Integrals Over Regions: Type I

Suppose that

$$E = \{(x,y,z) : (x,y) \in D,\, u_1(x,y) \le z \le u_2(x,y)\}.$$



$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\right] dA$$

Find $\iiint_E y\,dV$ if E is the region over

$$D = \{0 \le x \le 3,\ 0 \le y \le x\}$$

where for each (x,y),

$$x - y \le z \le x + y$$

Practice with Iterated Integrals

1. Find $\int_0^1 \int_y^{2y} \int_0^{x+y} 6xy \, dz \, dx \, dy$

2. Find $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} \, dx \, dz \, dy$

Integrals over Regions: Type II

If

$$E = \{(x,y,z) : (y,z) \in D,\ u_1(y,z) \leq x \leq u_2(y,z)\}$$

then

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\,dx\right] dA$$

Find $\iiint_E \dfrac{z}{x^2+z^2}\,dV$ if

$$E = \{(x,y,z) : 1 \leq y \leq 4,\ y \leq z \leq 4,\ 0 \leq x \leq z\}.$$

Integrals over Regions: Type III

If

$$E = \{(x, y, z) : (x, z) \in D,\ u_1(x, z) \leq y \leq u_2(x, z)\}$$

then

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\, dy \right] dA$$

Find $\iiint_E \sqrt{x^2 + z^2}\, dV$ if E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$