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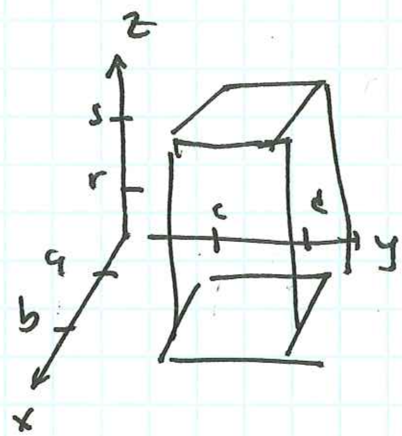
Goal: Define $\iiint_B f(x,y,z)\,dV$



$B = \{(x,y,z) : 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 3\}$

$$\iiint_B (xy + z^2)\,dV =$$

$$\int_0^3 \left(\int_0^1 \left(\int_0^2 (xy + z^2)\,dx \right) dy \right) dz =$$

$$\int_0^3 \left(\int_0^1 \left(\left[\frac{x^2 y}{2} + x z^2 \right] \Bigg|_{x=0}^{x=2} \right) dy \right) dz =$$

$$\int_0^3 \left(\int_0^1 \left(2\frac{4y}{4} + 2z^2 \right) dy \right) dz =$$

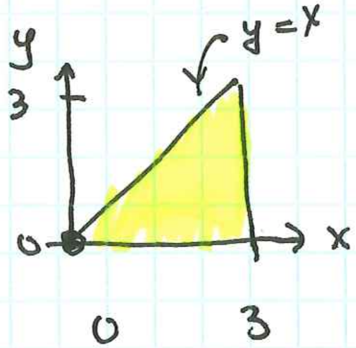
$$\int_0^3 \left(\left[y^2 + 2z^2 y \right]\Big|_{y=0}^{y=1} \right) dz =$$

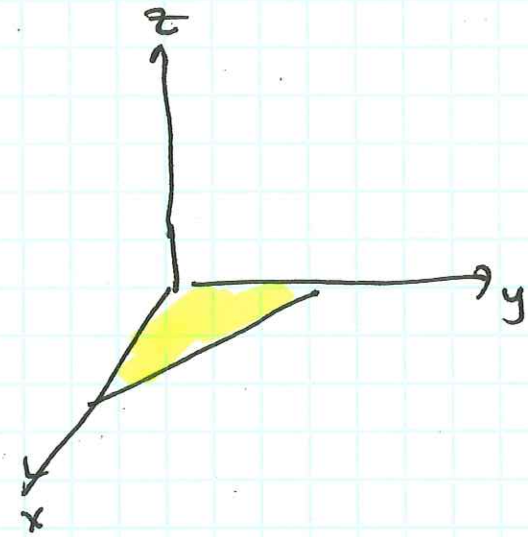
$$\int_0^3 \left(1 + 2z^2 \right) dz =$$

$$\left[z + \frac{2z^3}{3} \right]\Big|_0^3 =$$

$$3 + \frac{2 \cdot 3^3}{3} = 3 + 2 \cdot 9 = 21$$

$E = \{ 0 \le x \le 3,\ 0 \le y \le x,\ x-y \le z \le x+y \}$





$u_1(x,y) = x-y$

$u_2(x,y) = x+y$

u_1: $z = x-y$ or $x-y-z=0$

$\underset{\sim}{n} = \langle 1,-1,-1\rangle$

$u_2 = z = x+y$ $x+y-z=0$

$\underset{\sim}{n} = \langle 1,1,-1\rangle$

$\iiint_E y\,dV =$

$$\int_0^3 \left(\int_0^x \left(\int_{x-y}^{x+y} y\,dz \right) dy \right) dx =$$

$$\int_0^3 \left(\int_0^x \left(yz \Big|_{\cancel{x-y}\ z=x-y}^{\cancel{x+y}\ z=x+y} \right) dy \right) dx =$$

$$\int_0^3 \left(\int_0^x \left[y(x+y) - y(x-y) \right] dy \right) dx =$$

$$\int_0^3 \left(\int_0^x \left[\cancel{xy} + y^2 - \cancel{xy} + y^2 \right] dy \right) dx =$$

$$\int_0^3 \left(\int_0^x 2y^2 \, dy \right) dx =$$

$$\int_0^3 \left(\left[\frac{2y^3}{3} \right]_{y=0}^{y=x} \right) dx =$$

$$\int_0^3 \left[\frac{2x^3}{3} \right] dx = \frac{2x^4}{3 \cdot 4} \Big|_0^3$$

$$= \frac{2 \cdot 3^4}{3 \cdot 4} = \_\_\_\_$$

(1) $\int_0^1 \left(\int_y^{2y} \left(\int_0^{x+y} 6xy \, dz \right) dx \right) dy =$

$\int_0^1 \left(\int_y^{2y} \left[6xyz \right]_{z=0}^{z=x+y} dx \right) dy =$

$\int_0^1 \left(\int_y^{2y} \left[6xy(x+y) \right] dx \right) dy =$

$\int_0^1 \left(\int_y^{2y} \left(6x^2y + 6xy^2 \right) dx \right) dy =$

$\int_0^1 \left(\left[\frac{\cancel{6}^{\,2}x^3}{\cancel{3}} y + 3x^2y^2 \right] \Bigg|_{x=y}^{x=2y} \right) dy =$

$\int_0^1 \left[\left(16y^4 + \cancel{6}\,12y^4 \right) - 5y^4 \right] dy$

Type II

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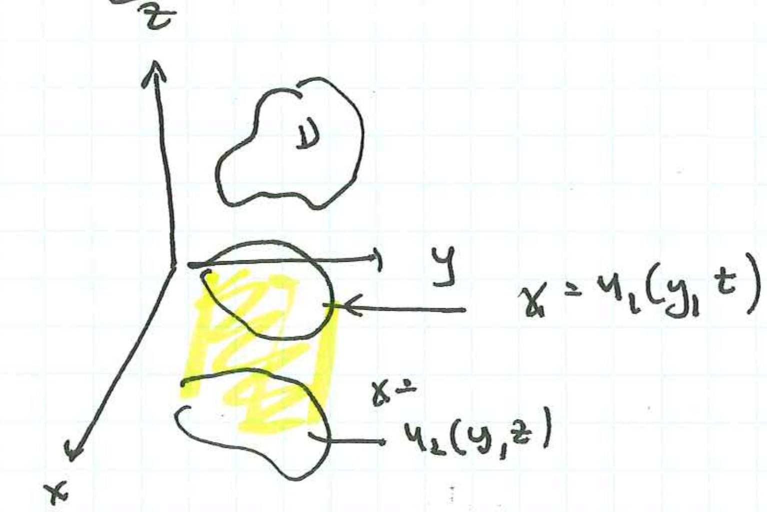
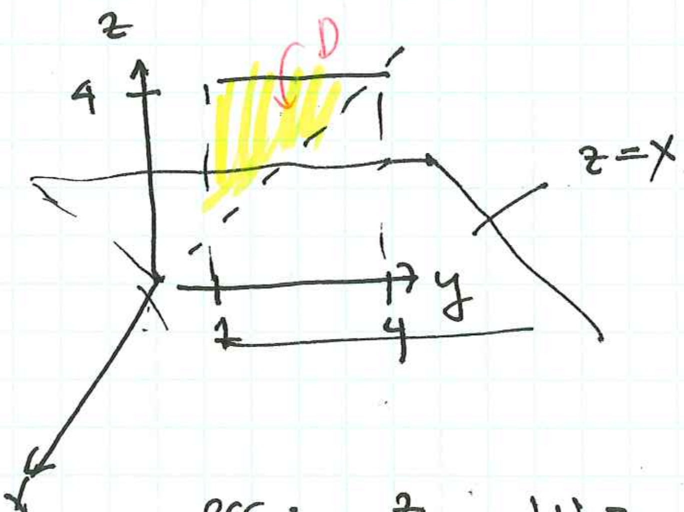


Example:

$$E = \{(x, y, z) : 1 \le y \le 4,\ y \le z \le 4,\ 0 \le x \le z\}$$



$$\iiint_E \frac{z}{x^2+z^2}\, dV =$$

$$= \int_1^4 \left(\int_y^4 \left(\int_0^z \frac{z}{x^2+z^2}\, dx \right) dz \right) dy$$

$$= \int_1^4 \left(\int_y^4 \left(\frac{\pi}{4} \right) dz \right) dy$$

Note: $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$

$$\int_0^z \frac{z \, dx}{x^2+z^2} = z \cdot \int_0^z \frac{1}{x^2+z^2} \, dx$$

$$= \cancel{z} \cdot \frac{1}{\cancel{z}} \tan^{-1}\left(\frac{x}{z}\right) \Big|_{x=0}^{x=z}$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4} \int_1^4 \left(\int_y^4 dz \right) dy$$

$$= \frac{\pi}{4} \int_1^4 \cancel{(y-4)\, dy} \; (4-y) \, dy$$

$$= \frac{\pi}{4} \left(4y - y^2/2 \right) \Big|_{y=1}^{y=4}$$

Type III:

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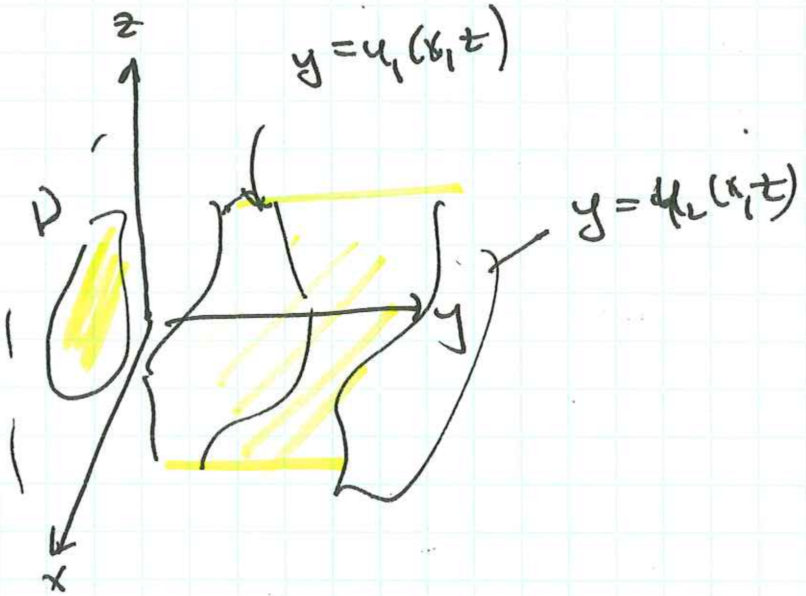
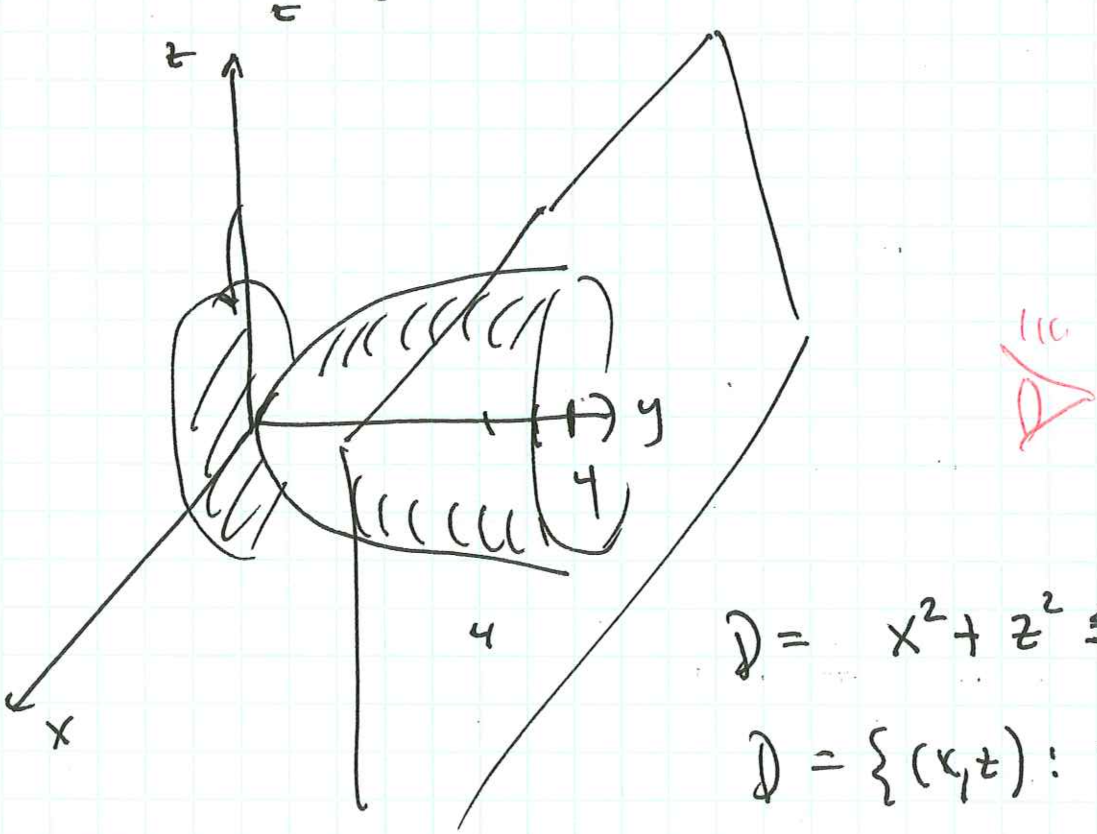


Type III Ex:

~~$E = \{ (x, y, z) : y = x^2 + z^2$ and~~



$D = x^2 + z^2 \leq 4$

$D = \{ (x, z) : x^2 + z^2 \leq 4 \}$

$$\iiint_E \sqrt{x^2+z^2}\, dV =$$

$$\iint_D \left(\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\, dy \right) dA$$