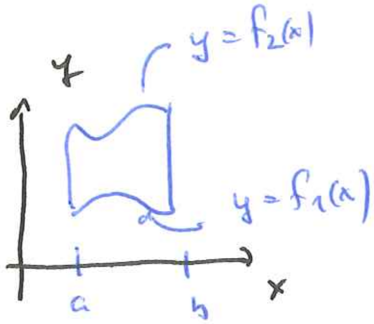
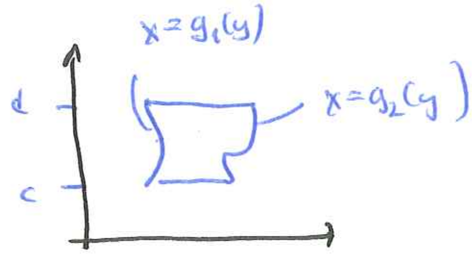




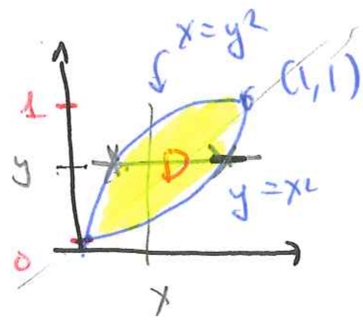
Type I

$$\int_a^b \left(\int_{f_1(x)}^{f_2(x)} \cdots \, dy \right) dx$$

Type II

$$\int_c^d \left(\int_{g_1(y)}^{g_2(y)} \cdots \, dx \right) dy$$

Double Integrals



$$D = \{ (x,y) : 0 \le y \le 1, \\ y^2 \le x \le \sqrt{y} \}$$

Type II: $\displaystyle \int_0^1 \left(\int_{y^2}^{\sqrt{y}} xy^2 \, dx \right) dy$

$$= \int_0^1 \left(y^2 \left(\frac{x^2}{2} \Big|_{y^2}^{\sqrt{y}} \right) \right) dy$$

$$= \int_0^1 \left(y^2 \cdot \left[\frac{y}{2} - \frac{y^4}{2} \right] \right) dy$$

$$= \int_0^1 \left(\frac{y^3}{2} - \frac{y^6}{2} \right) dy = \left[\frac{y^4}{8} - \frac{y^7}{14} \right]_{y=0}^{y=1}$$

Polar

$1 \le r \le 2 \qquad \frac{\pi}{2} \le \theta \le 2\pi$

$$\iint_D xy \, dA = \int_{\pi/2}^{2\pi} \left(\int_1^2 \underline{r^2 \cos\theta \sin\theta} \, \boxed{r} \, dr \right) d\theta$$

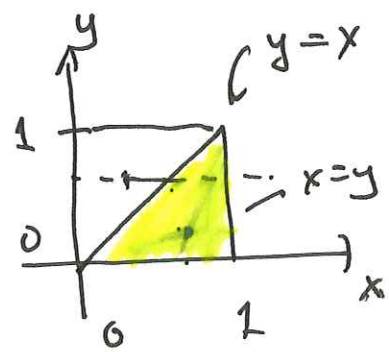
$x = r\cos\theta$

$y = r\sin\theta$

$$\int_0^1 \left(\int_y^1 \left(\int_0^y \square \, dz \right) dx \right) dy =$$

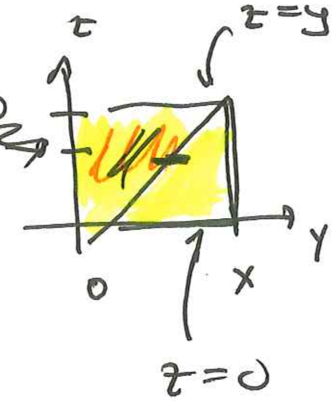
$$\underline{\int_0^1 \int_y^1} \left[\int_0^y \square \, dz \right] dx \, dy =$$

$$\int_0^1 \left(\int_0^x \left(\int_0^y \square \, dz \right) dy \right) dx$$



$$\int_0^1 \left[\int_0^{x} \int_0^{y} f(x,y,z)\, dz\, dy \right] dx$$

$$= \int_0^1 \left[\int_0^x \left(\int_z^x f(x,y,z)\, dy \right) dz \right] dx$$



Cylindrical Coordinates

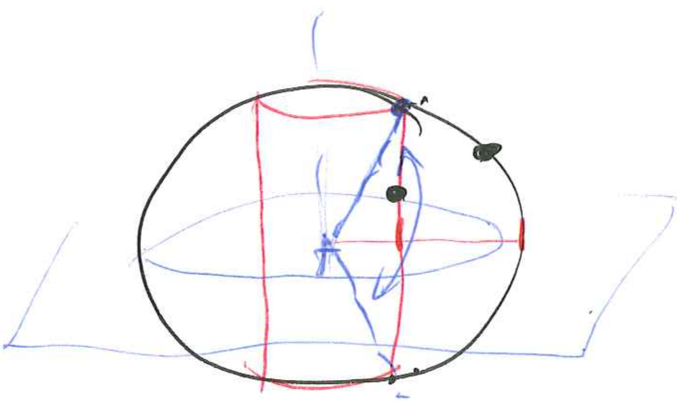
$$dV = r\, dr\, d\theta\, dz$$

$$E = \{ (r, \theta, z) : \underline{1} \le r \le \underline{2},\ \underline{0} \le \theta \le \underline{2\pi},\ \underline{0} \le z \le \underline{r\cos\theta + 2} \}$$

$$\int_0^{2\pi} \int_1^2 \left(\int_0^{r\cos\theta+2} r \sin\theta\ \cancel{r\, dr)\, d\theta)}\ dz \right) r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_1^2 \left(\int_0^{r\cos\theta + 2} r\sin\theta \, dz \right) r\,dr\,d\theta$$

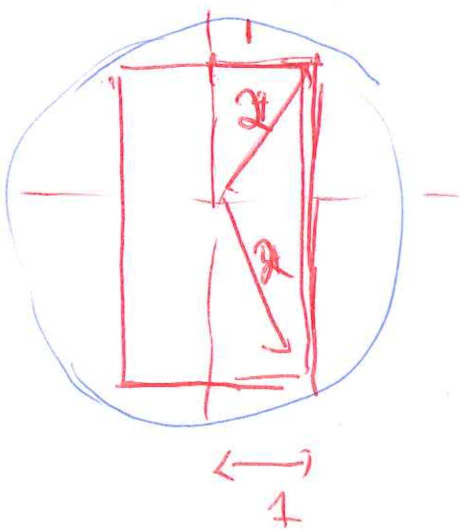
Spherical coordinates

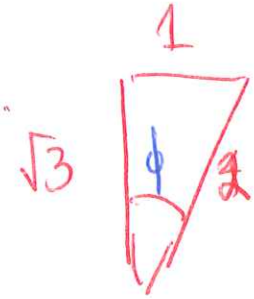


$$\frac{1}{\sin\phi} \le \rho \le 2$$

$$0 \le \theta \le 2\pi$$

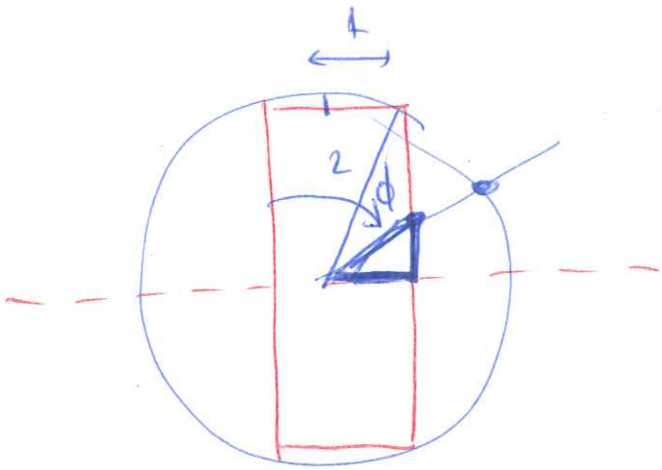
$$\frac{\pi}{6} \le \phi \le 5\pi/6$$

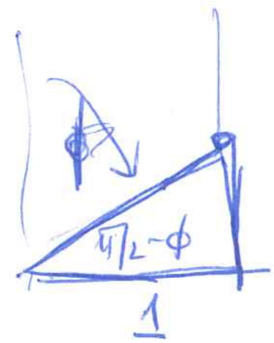


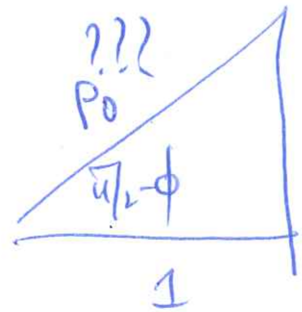


$\phi = \frac{\pi}{6}$

$\tan\phi = \frac{1}{\sqrt{3}}$







$$\cos(\pi/2 - \phi) = \frac{1}{\rho_0}$$

$$\sin\phi = \frac{1}{\rho_0}$$

$$\therefore \rho_0 = \frac{1}{\sin\phi}$$

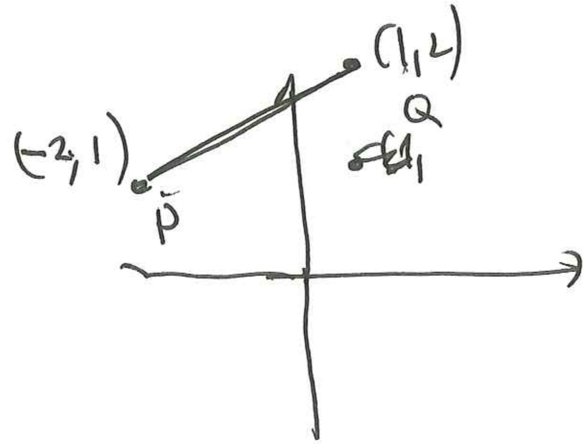
$$\frac{1}{\sin\phi} \le \rho \le 2$$

$$0 \le \theta \le 2\pi$$

$$\pi/6 \le \phi \le 5\pi/6$$

$$V = \int_0^{2\pi} \left(\int_{\pi/6}^{5\pi/6} \left(\int_{1/\sin\phi}^{2} 1 \; \underbrace{\rho^2 \, d\rho}_{\sin\phi} \right) d\phi \right) d\theta$$

Line Integrals



$$\vec{PQ} = \langle 3, 1 \rangle$$

$$\vec{r}(t) = \langle -2, 1 \rangle + t\langle 3, 1 \rangle$$

$$r(0) = \langle -2, 1 \rangle$$

$$r(1) = \langle 1, 2 \rangle$$

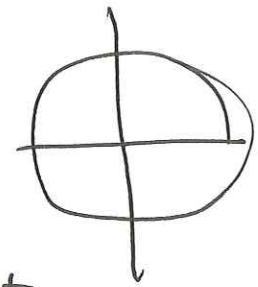
$$x(t) = -2 + 3t \qquad x'(t) = 3$$

$$y(t) = 1 + t \qquad y'(t) = 1$$

$$ds = \sqrt{3^2 + 1^2}\, dt = \sqrt{10}\, dt$$

$$\int_C 4x^3\, ds = \int_0^1 4(-2+3t)^3 \sqrt{10}\, dt$$

Line Integrals ctd



$x(t) = 2\cos t$ $\quad x'(t) = -2\sin t$

$y(t) = 2\sin t$ $\quad y'(t) = 2\cos t$

$0 \le t \le 2\pi$

$ds = \sqrt{x'(t)^2 + y'(t)^2}\, dt$

$= \sqrt{4\sin^2 t + 4\cos^2 t}$

$= 2\, dt$

$\int_C xy\, ds =$

$$\int_0^{2\pi} (2\cos t)(2\sin t)\, 2\, dt$$

Change of Variables

$R: \quad x^2 - xy + y^2 \le 2$

$x = \sqrt{2}u - \sqrt{\frac{2}{3}}v$

$y = \sqrt{2}u + \sqrt{\frac{2}{3}}v$

$J = \begin{vmatrix} \sqrt{2} & -\sqrt{\frac{2}{3}} \\ \sqrt{2} & \sqrt{\frac{2}{3}} \end{vmatrix} = \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

$x^2 = 2u^2 - \frac{2 \cdot 2}{\sqrt{3}}uv + \frac{2}{3}v^2$

$-\ xy = -(2u^2 - \frac{2}{3}v^2)$

$y^2 = 2u^2 + \frac{2 \cdot 2}{\sqrt{3}}uv + \frac{2}{3}v^2$

$x^2 - xy + y^2 = 2u^2 + 2v^2 \le 2$

$u^2 + v^2 \le 1$

$\iint_R x^2 - xy + y^2 \, dA =$

$\iint_S 2(u^2+v^2) \cdot \frac{4}{\sqrt{3}} \cdot du\, dv$

S is a disc of radius 1

$$\iint_S 2(u^2+v^2)\cdot\frac{4}{\sqrt{3}}\,du\,dv$$

$u = r\cos\theta$

$v = r\sin\theta$

$$= \int_0^{2\pi}\int_0^1 2r^2\cdot\frac{4}{\sqrt{3}}\,r\,dr\,d\theta$$