

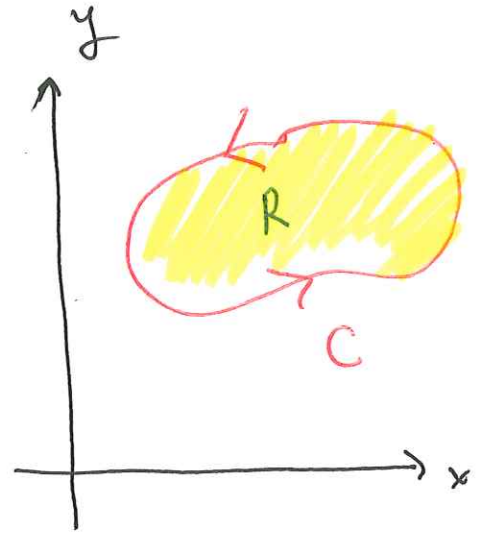
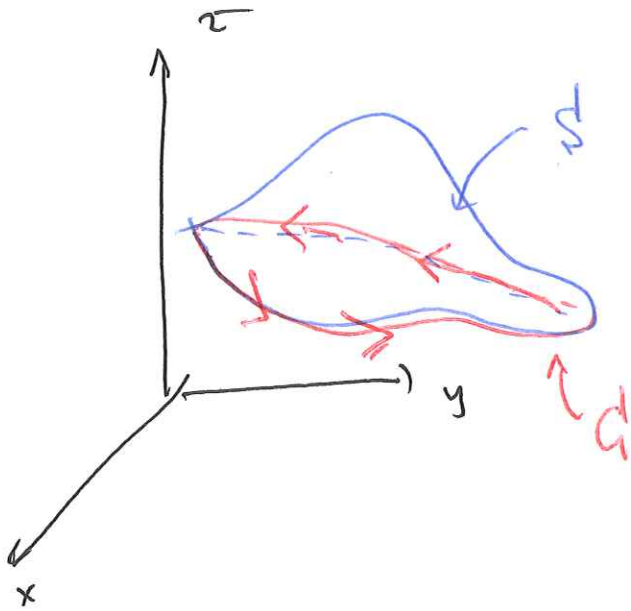
Office Hours Today

4-5 PM

755 POT

Stokes

Green



$$\vec{F}(x, y, z) = P(x, y, z) \hat{i} + Q(x, y, z) \hat{j} + R(x, y, z) \hat{k}$$

$$\vec{F}(x, y) = P(x, y) \hat{i} + Q(x, y) \hat{j}$$

$$\int_C \vec{F} \cdot d\vec{r} =$$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot \frac{d\vec{S}}{ds} ds$$

②

$$\int_C \vec{F} \cdot d\vec{r} =$$

$$\int_C P dx + Q dy$$

Since

$$\int \vec{F}(x(t), y(t)) \cdot (x'(t)\hat{i} + y'(t)\hat{j}) dt =$$

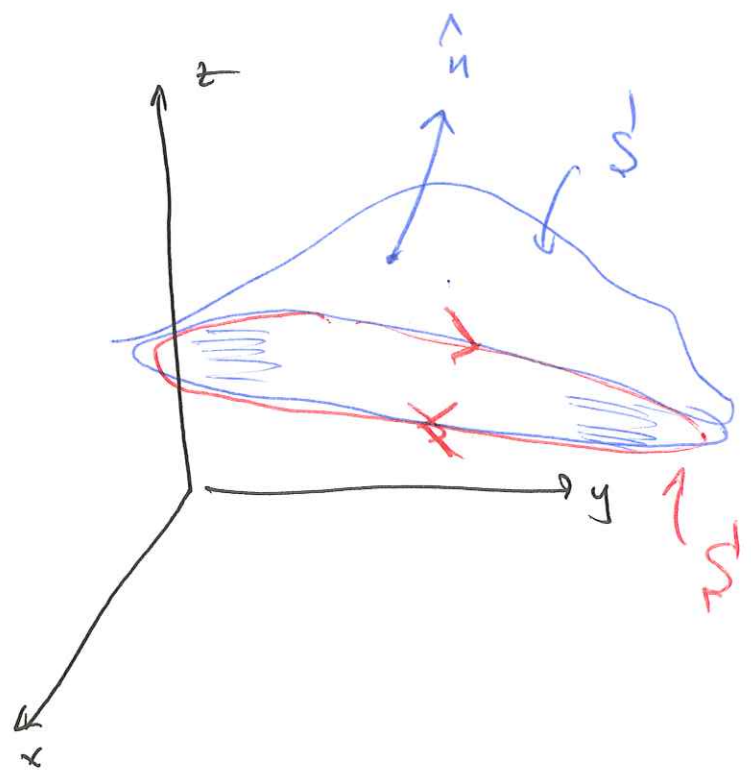
$$\int (P(x(t), y(t))\hat{i} + Q(x(t), y(t))\hat{j}) \cdot$$

$$(x'(t)\hat{i} + y'(t)\hat{j}) dt =$$

$$\int P(x(t), y(t)) x'(t) dt +$$

$$\int Q(x(t), y(t)) y'(t) dt =$$

$$\int P dx + \int Q dy$$



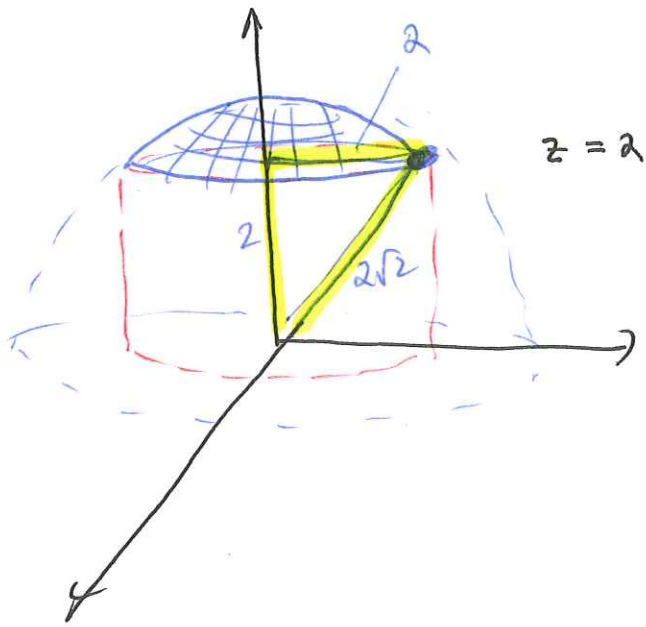
$$f = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$\vec{\nabla} \times \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$= \hat{i} \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) +$$

$$\hat{j} \left(\frac{\partial R}{\partial z} - \frac{\partial P}{\partial x} \right) +$$

$$\hat{k} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)$$



$$x^2 + y^2 + z^2 = 8$$

$$x^2 + y^2 = 4$$

Intersection:

$$\underbrace{x^2 + y^2}_{4} + z^2 = 8$$

$$4$$

$$4 + z^2 = 8$$

$$z^2 = 4$$

$$z = 2$$

Circle of intersection:

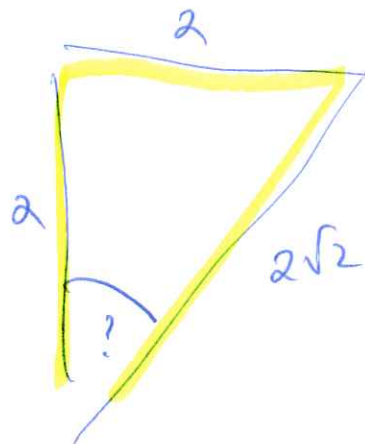
$$\begin{cases} z = 2 \\ x^2 + y^2 = 4 \end{cases}$$

Parameterize in spherical coords!

$$\rho = 2\sqrt{2}$$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq \varphi \leq \pi/4$$



(5)

$$\vec{F}(x, y, z) = x^2 \sin z \hat{i} + y^2 \hat{j} + xy \hat{k}$$

C: Intersection of $z = 1 - x^2 - y^2$ and xy plane

$$z = 0$$

$$\therefore 0 = 1 - x^2 - y^2$$

$$\therefore x^2 + y^2 = 1$$

$$C: \vec{r}(t) = \langle \cos t, \sin t, 0 \rangle \quad 0 \leq t \leq 2\pi$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \langle 0, \sin^2 t, \cos t \sin t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle dt$$

$$= \int_0^{2\pi} \sin^2 t \cos t dt$$

$$= \frac{\sin^3 t}{3} \Big|_0^{2\pi} = 0$$

$$u = \sin t$$

$$du = \cos t dt$$

$$u = \sin t$$

$$\int \sin^2 t \cos t dt = \int u^2 du$$

$$= \frac{u^3}{3} + C$$

(5)

$$\int_C \vec{F} \cdot d\vec{r} =$$

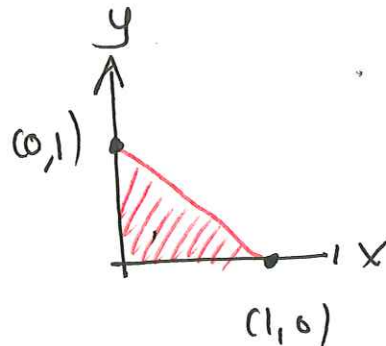
$$\iint_{S'} (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$$

$$\vec{\nabla} \times \vec{F} = \langle -2z, -2x, -2y \rangle$$

The surface S' is part of the plane.

$$\boxed{x + y + z = 1}$$

that sits over
a triangle in the
 xy plane.



Parameterize by

$$\vec{r}(x, y) = \langle x, y, 1 - x - y \rangle$$

for x, y range over the triangle.

$$\vec{r}_x(x, y) = \langle 1, 0, -1 \rangle$$

$$\vec{r}_y(x, y) = \langle 0, 1, -1 \rangle$$

$$\boxed{\begin{array}{l} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 - x \end{array}}$$

(7)

$$\begin{aligned}\vec{r}_x \times \vec{r}_y &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} \\ &= \hat{i} + \hat{j} + \hat{k} \\ &= \langle 1, 1, 1 \rangle\end{aligned}$$

S : lies over $\bar{D} = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1-x\}$

$$\begin{aligned}\vec{r}_x \times \vec{r}_y &= \langle 1, 1, 1 \rangle \\ \vec{\nabla}_x P &= \langle -2z, -2x, -2y \rangle \quad z = 1-x-y \\ &= \langle -2(1-x-y), -2x, -2y \rangle\end{aligned}$$

$$\begin{aligned}\iint_S \vec{\nabla}_x P \cdot d\vec{S} &= \\ &= \int_0^1 \left(\int_0^{1-x} \langle -2(1-x-y), -2x, -2y \rangle \cdot \langle 1, 1, 1 \rangle dy \right) dx \\ &= \int_0^1 \left(\int_0^{1-x} (-2(1-x-y) - 2x - 2y) dy \right) dx\end{aligned}$$

$$= \int_0^1 \left(\int_0^{1-x} -2 \, dy \right) dx$$

$$= -2 \left(\frac{1}{2} \right) \text{ area of } \img alt="A small diagram of a right-angled triangle with red diagonal hatching. The triangle is formed by a vertical line, a horizontal line, and a hypotenuse connecting the top of the vertical line to the right end of the horizontal line." data-bbox="615 261 728 320"/>$$

$$= -1$$