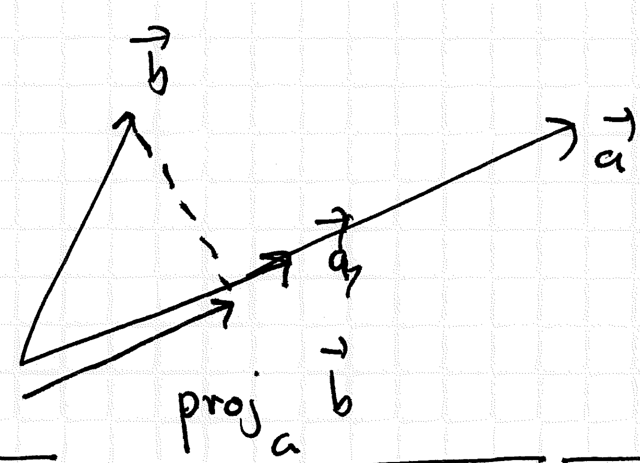
12.3 Dot Products

Projections (later)



Projection of $\vec{b}$ onto $\vec{a}$ (application of dot product)

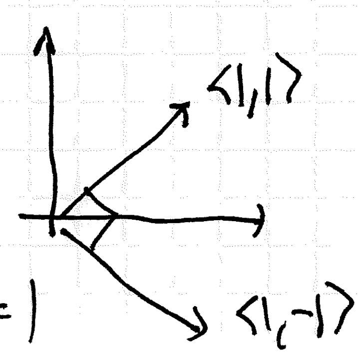
$\vec{a}$ $\langle 1, 2, 1 \rangle$ $\vec{b} = \langle 3, -2, 5 \rangle$

$$\vec{a} \cdot \vec{b} = 1 \cdot 3 + 2(-2) + 1 \cdot 5$$
$$= 3 - 4 + 5$$
$$= 4$$

Examples from slide:

③ $\vec{a} = \langle 3, -4, 1 \rangle$ $\vec{b} = \langle 2, 5, 0 \rangle$

① $\langle 1, 1 \rangle \cdot \langle 1, -1 \rangle = 0$



② $a = b = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$$a \cdot b = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

Fill in the blanks

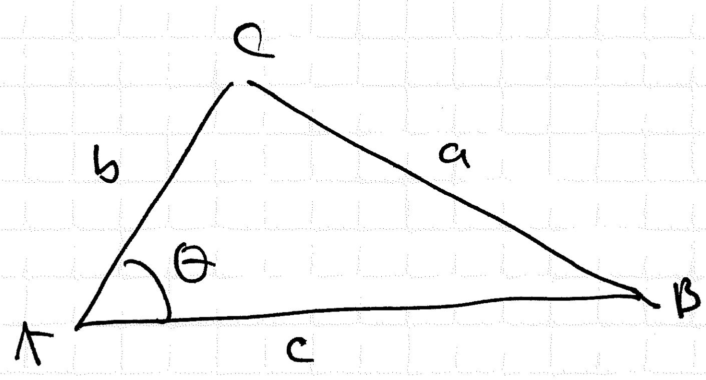
$$\vec{a}\cdot\vec{a} = \langle a_1, a_2, a_3\rangle \cdot \langle a_1, a_2, a_3\rangle$$
$$= a_1^2 + a_2^2 + a_3^2$$
$$= |\vec{a}|^2$$

$$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$$

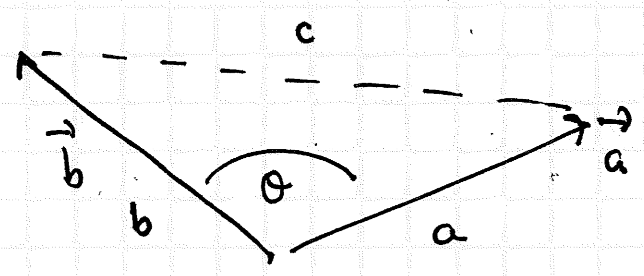
$$(c\vec{a})\cdot\vec{b} = \underline{c}\,(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot c\vec{b})$$

$$\vec{0}\cdot\vec{a} = 0$$

Law of Cosines



$c^2 = a^2 + b^2$
$- 2ab\cos\theta$



$$c = |\vec{b} - \vec{a}|$$

$$c^2 = |\vec{b} - \vec{a}|^2 = (\vec{b} - \vec{a}) \cdot (\vec{b} - \vec{a})$$

$$= \vec{b} \cdot \vec{b} - \vec{b} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{a}$$

$$= |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{a}|^2$$

(1) $c^2 = a^2 + b^2 - 2\vec{a} \cdot \vec{b}$

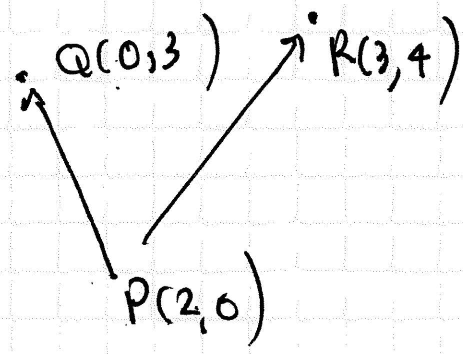
(2) $c^2 = a^2 + b^2 - 2ab\cos\theta$

$$\therefore 2\vec{a} \cdot \vec{b} = 2ab\cos\theta$$

$\langle 1,1,1\rangle \quad \langle 2,2,2\rangle \quad \langle -5,-5,-5\rangle \quad \parallel$

① $\langle 9,3\rangle \cdot \langle -2,6\rangle = 9\cdot(-2) + 3\cdot 6 = 0$

②



$\vec{PQ} = \langle -2,3\rangle$

$\vec{PR} = \langle 1,4\rangle$

$\vec{PQ}\cdot\vec{PR} = -2+12 = 10$

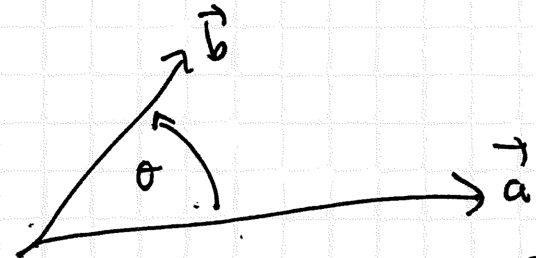
$|\vec{PQ}| = \cancel{\sqrt{2^2+9^2} = \sqrt{97}}\ \sqrt{4+9}$ $\quad \begin{array}{r} 16 \\ 81 \\ \hline 97 \end{array}$

$|\vec{PR}| = \sqrt{1+16} = \sqrt{17}$

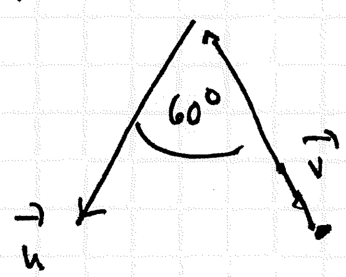
$$\cos\theta = \frac{10}{\cancel{\sqrt{97}}\sqrt{17}} = \frac{10}{\sqrt{13}\sqrt{17}}$$

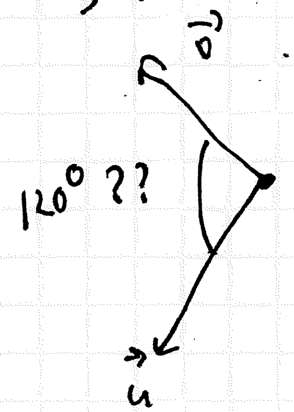
$\sqrt{13}$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$



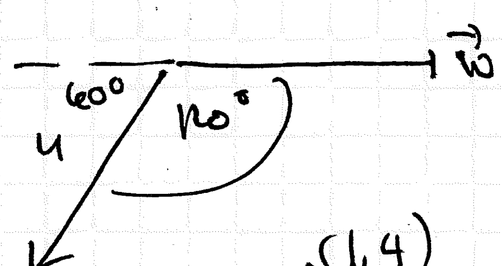
Triangle:





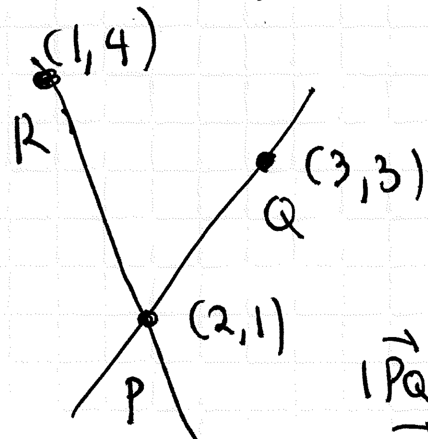
$$\vec{u} \cdot \vec{v} = 1 \cdot 1 \cdot \cos(120^\circ)$$

$$= -\frac{1}{2}$$



$$\vec{u} \cdot \vec{w} = -\frac{1}{2}$$

Lines



$\overrightarrow{PQ} = \langle 1, 2 \rangle$

$\overrightarrow{PR} = \langle -1, 3 \rangle$

$|\overrightarrow{PQ}| = \sqrt{5}$ $\qquad \overrightarrow{PQ} \cdot \overrightarrow{PR} = 5$

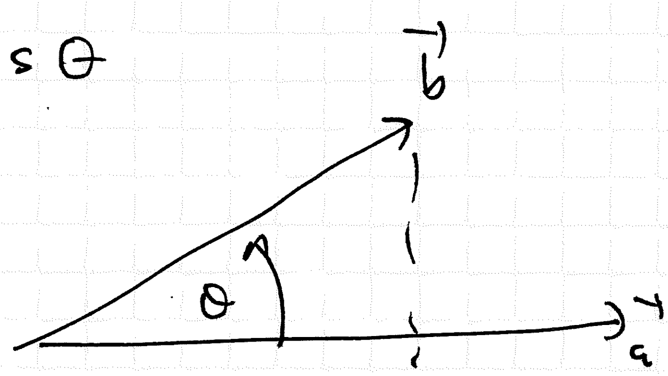
$|\overrightarrow{PR}| = \sqrt{10}$

$$\cos\theta = \frac{5}{\sqrt{5}\cdot\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{1}{2}} =$$

$$\theta = \frac{\pi}{4} \quad \text{or} \quad 45^{\circ}$$

$$\text{comp}_{\vec{a}}\,\vec{b} = |\vec{b}|\cos\theta$$

$$=$$



$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = |\vec{b}|\cos\theta = \text{comp}_{\vec{a}}\,\vec{b}$$

$\text{proj}_{\vec{a}} \vec{b}$ $\quad \vec{a} = \langle -5, 12 \rangle$

$\vec{b} = \langle 4, 6 \rangle$

$|\vec{a}| = \sqrt{25 + 144} = 13$

$\vec{a} \cdot \vec{b} = -20 + 72 = 52$

$\text{comp}_{\vec{a}} \vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \dfrac{52}{13}$

$\text{proj}_{\vec{a}} \vec{b} = \dfrac{52}{13} \cdot \dfrac{1}{13} \langle -5, 12 \rangle$