Math 213 - Equations of Lines and Planes, I

Peter A. Perry

University of Kentucky

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Reminders			

- Access your WebWork account only through Canvas!
- Homework A2 on Sections 12.3-12.4 is due tonight!
- Applications for an alternate Exam 1 were due Wednesday! Review your schedule and apply for all alternate exams at once by using the Google Form linked from Canvas or the course home page.



Unit I: Geometry and Motion in Space

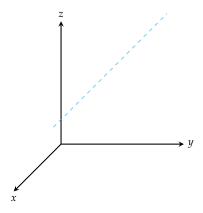
- 12.1 Lecture 1: Three-Dimensional Coordinate Systems
- 12.2 Lecture 2: Vectors in the Plane and in Space
- 12.3 Lecture 3: The Dot Product
- 12.4 Lecture 4: The Cross Product
- 12.5 Lecture 5: Equations of Lines and Planes, I
- 12.5 Lecture 6: Equations of Lines and Planes, II
- 12.6 Lecture 7: Surfaces in Space
- 13.1 Lecture 8: Vector Functions and Space Curves
- 13.2 Lecture 9 Derivatives and Integrals of Vector Functions Lecture 10: Exam I Review

		Review
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Learning Goals

- Learn how to write the parametric equation of a line
- Learn how to write the symmetric equation of a line
- Learn how to write the vector equation of a plane
- Learn how to write the scalar equation of a plane





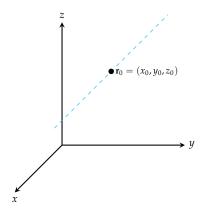
A line *L* in three-dimensional space is determined by

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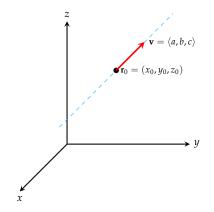


A line *L* in three-dimensional space is determined by

• A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line

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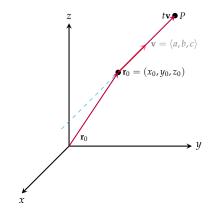


A line *L* in three-dimensional space is determined by

- A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line
- A vector **v** = $\langle a, b, c \rangle$ that gives the direction of the line

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A line *L* in three-dimensional space is determined by

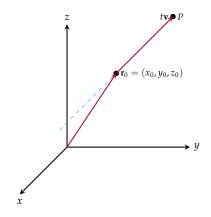
- A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line
- A vector **v** = $\langle a, b, c \rangle$ that gives the direction of the line

Any point *P* on the line can be expressed as

 $\mathbf{r}_0 + t\mathbf{v}$

for some real number *t* called the *pa-rameter*





If

$$\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle, \quad \mathbf{v} = \langle a, b, c \rangle,$$

the *function*

 $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$

traces out a line through

 $P = (x_0, y_0, z_0)$

in the direction of

$$\mathbf{v} = \langle a, b, c \rangle$$

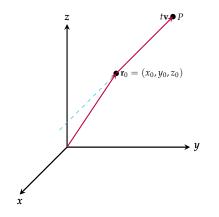
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 Line - Parametric
 Line - Symmetric
 Plane - Vector
 Plane - Scalar
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Line - Parametric Equation



If $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ then $x(t) = x_0 + at$ $y(t) = y_0 + bt$ $z(t) = z_0 + ct$

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Line - Parametric Equation

If $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ then

 $x(t) = x_0 + at$ $y(t) = y_0 + bt$ $z(t) = z_0 + ct$

gives the parametric equations for a line through $P(x_0, y_0, z_0)$ in direction $\langle a, b, c \rangle$

- **1** Find the parametric equations of a line *L* through the points P(1, 2, -1) and Q(2, 3, 4).
- **2** Find the parametric equations of the line *L* through the point (1, 2, 3) and parallel to the vector $\langle 2, -3, 4 \rangle$

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 Line - Parametric
 Line - Symmetric
 Plane - Vector
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Line - Symmetric Equation

If we begin with the parametric equations of a line:

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$



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Line - Parametric Line - Symmetric Plane - Vector Plane - Scalar Review

Line - Symmetric Equation

If we begin with the parametric equations of a line:

 $\begin{aligned} x(t) &= x_0 + at \\ y(t) &= y_0 + bt \\ z(t) &= z_0 + ct \end{aligned}$

we can eliminate the parameter to get the symmetric equation of a line;

 $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

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The numbers (*a*, *b*, *c*) are the *direction numbers* of the line.

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Line - Parametric Line - Symmetric Plane - Vector Plane - Scalar Review
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Line - Symmetric Equation

If we begin with the parametric equations of a line:

 $x(t) = x_0 + at$ $y(t) = y_0 + bt$ $z(t) = z_0 + ct$

we can eliminate the parameter to get the symmetric equation of a line;

 $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

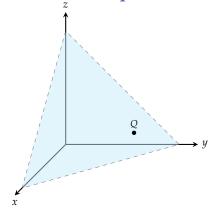
The numbers (*a*, *b*, *c*) are the *direction numbers* of the line.

- **1** Find the parametric and symmetric equations of the line through the origin and the point (4, 3, -1)
- **2** Find the parametric and symmetric equations of the line through (2, 1, 0) and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

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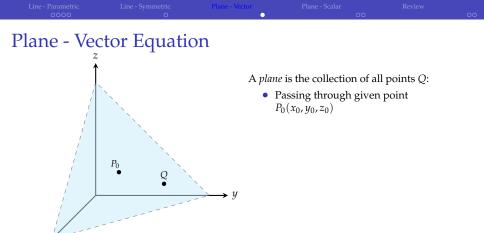


Plane - Vector Equation



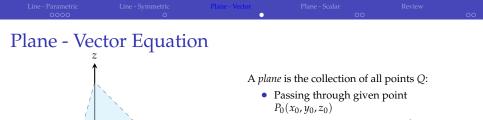
A *plane* is the collection of all points *Q*:

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 P_0

• Having the property that $\overline{P_0Q}$ is perpendicular to a vector **n**, the *normal vector*

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• Having the property that $\overline{P_0Q}$ is perpendicular to a vector **n**, the *normal vector*

That is

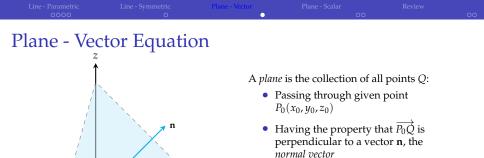
$$\mathbf{n}\cdot\overrightarrow{P_0Q}=0$$

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x

Math 213 - Equations of Lines and Planes

 P_0



f
$$\mathbf{r}_0 = \overrightarrow{OP_0}, \mathbf{r} = \overrightarrow{OQ},$$
 then...

 $\mathbf{n} \cdot \overrightarrow{P_0 Q} = 0$

That is

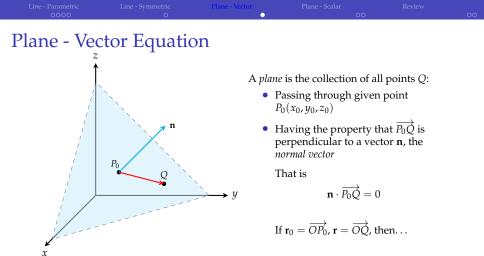
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$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$
 OR $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

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n

 P_0

• Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the *normal vector* That is

$$\mathbf{n}\cdot\overrightarrow{P_0Q}=0$$

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x



n

 P_0

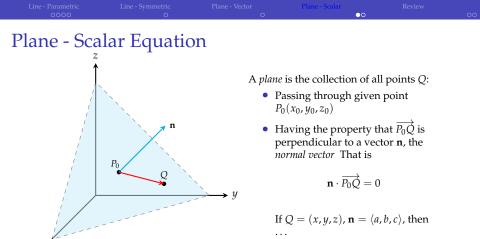
• Having the property that $\overline{P_0Q}$ is perpendicular to a vector **n**, the *normal vector* That is

$$\mathbf{n}\cdot\overrightarrow{P_0Q}=0$$

If
$$Q = (x, y, z)$$
, $\mathbf{n} = \langle a, b, c \rangle$, then ...

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$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

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Let

$$P_0 = P_0(x_0, y_0, z_0), \quad \mathbf{n} = \langle a, b, c \rangle, \quad P = P(x, y, z)$$

The **vector equation** of the plane through P_0 with normal **n** is

 $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

where **r** and **r**₀ are position vectors for *P* and *P*₀ respectively. The **scalar equation** of the plane through *P*₀ with normal **n** is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

- **1** Find the vector equation of a plane through the origin and perpendicular to the vector $\langle -1, 2, 5 \rangle$
- **2** Find the scalar equation of the plane through (1, -1, -1) and parallel to the plane 5x y z = 6

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(3) Find the equation of the plane that contains the line x = 1 + t, y = 2 - t, z = 4 - 3t and is parallel to the plane 5x + 2y + z = 1

		Review	
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Summary

- We learned that a line is determined by a point $P_0 = (x_0, y_0, z_0)$ on the line, and a vector $\mathbf{v} = \langle a, b, c \rangle$ that points along the line
 - The *parametric* equations of a line are

$$x(t) = x_0 + at$$
, $y(t) = y_0 + bt$, $z(t) = z_0 + ct$

• The symmetric equations of a line are

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

- We learned that a plane is determined by a point $P_0 = (x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n} = \langle a, b, c \rangle$ *normal* to the plane
 - The *vector* equation of a plane is

$$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0)=\mathbf{0}$$

• The scalar equation of a plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

		Review	
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Homework

- Webwork A2 is due tonight!
- Re-read section 12.5, pp. 823-830



Peter A. Perry