$$y = f(x) = e^{x^2+1}$$

$$y = f(x) = e^{u}$$

$$u(x) = x^2+1$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^{u} \cdot 2x \quad \Big\} \text{ substitute } u = x^2+1$$

$$= e^{x^2+1} \cdot (2x)$$

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1) $y = u^3 \qquad u = \cos x \qquad$ (so $y = (\cos x)^3$)

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2 \cdot (-\sin x) = 3 \cdot \cos^2 x\,(-\sin x)$$

2) $g(x) = (x^2+1)^{3/2} \qquad y = u^{3/2} \quad u = x^2+1$

$$g'(x) = \underbrace{3/2\,(x^2+1)^{1/2}}_{\frac{dy}{du}} \cdot \underbrace{2x}_{\frac{du}{dx}}$$

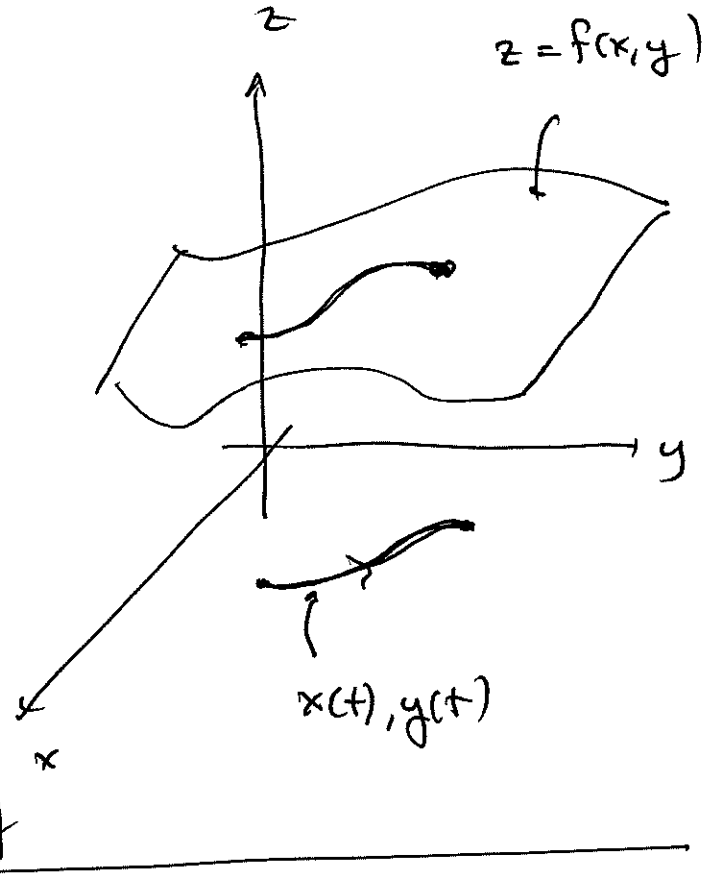
$z = f(x, y)$

$x = g(t)$

$y = h(t)$

$\frac{d}{dt} f(x(t), y(t)) =$

$$\boxed{\frac{dz}{dt} = \underbrace{\frac{\partial f}{\partial x}\frac{dx}{dt}}_{\text{change of } x} + \underbrace{\frac{\partial f}{\partial y}\frac{dy}{dt}}_{\text{change of } y}}$$



#1 $\quad z = \sin x \cos y \qquad x = \sqrt{t} \qquad y = 1/t$

$\frac{\partial f}{\partial x} = \cos x \cos y \qquad \frac{\partial f}{\partial y} = -\sin x \sin y$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dy}{dt} = -\frac{1}{t^2}$

$$\frac{dz}{dt} = \underbrace{\cos x \cos y \cdot \frac{1}{2\sqrt{t}}}_{} + \underbrace{-\sin x \sin y \cdot \left(-\frac{1}{t^2}\right)}_{}$$

$$\frac{dz}{dt} = \cos\sqrt{t} \cdot \cos(1/t) \cdot \frac{1}{2\sqrt{t}} \quad +$$

$$- \sin\sqrt{t} \, \sin\left(\frac{1}{t}\right) \cdot \left(-\frac{1}{t^2}\right)$$

#2

$$z = f(x,y) = \sqrt{1+xy}$$

$$x = g(t) = \tan t$$

$$y = h(t) = \arctan t$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{1+xy}}\, y \qquad \frac{\partial f}{\partial y} = \frac{1}{2\sqrt{1+xy}} \cdot x$$

$$\frac{dx}{dt} = \sec^2 t \qquad \frac{dy}{dt} = \frac{1}{1+t^2}$$

$$\frac{dz}{dt} = \frac{y}{2\sqrt{1+xy}} \sec^2 t + \frac{x}{2\sqrt{1+xy}} \cdot \frac{1}{1+t^2}$$

(Now substitute $x = \tan t$, $y = \arctan t$)

$$dz = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\,\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$$

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case 2:

$$z = f(x,y) \qquad \begin{aligned} x &= x(s,t) \\ y &= y(s,t) \end{aligned}$$

$$\frac{\partial z}{\partial s} = \cancel{\frac{\partial f}{\partial y}}\cdot\ \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\,\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

1/15/2019 (c)

#1 $z = \tan^{-1}(x^2+y^2)$

$x = s \ln t$

$y = te^t$

$$\frac{\partial f}{\partial x} = \frac{1}{1+(x^2+y^2)^2}(2x) \qquad \frac{\partial f}{\partial y} = \frac{1}{(1+(x^2+y^2)^2)}\cdot(2y)$$

$$\frac{\partial x}{\partial s} = \ln t \qquad \frac{\partial y}{\partial s} = 0$$

$$\frac{\partial x}{\partial t} = s\cdot\frac{1}{t} \qquad \frac{\partial y}{\partial t} = e^t + te^t = (1+t)e^t$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= \frac{2x}{(1+(x^2+y^2)^2)}\cdot \ln t + \frac{2y}{(1+(x^2+y^2)^2)}\cdot 0$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= \frac{2x}{(1+(x^2+y^2)^2)}\cdot\frac{s}{t} + \frac{2y}{(1+(x^2+y^2)^2)}\cdot(1+t)e^t$$

$$z = f(x,y) = \sqrt{x}\, e^{xy}$$

$$x = 1 + st \qquad y = s^2 - t^2$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x}} e^{xy} + y\sqrt{x} e^{xy} \qquad \frac{\partial f}{\partial y} = \sqrt{x} \cdot x e^{xy}$$

$$\frac{\partial x}{\partial s} = t \qquad \frac{\partial y}{\partial s} = 2s$$

$$\frac{\partial x}{\partial t} = s \qquad \frac{\partial y}{\partial t} = -2t$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= \left[\frac{1}{2\sqrt{x}} e^{xy} + y\sqrt{x} e^{xy} \right] \cdot t +$$

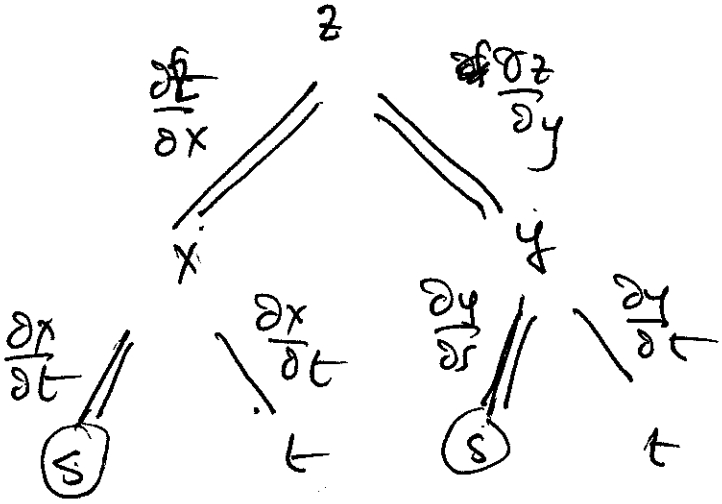
$$\left[x\sqrt{x} e^{xy} \right] \cdot (2s)$$

4/15/2019

$$z = f(x, y)$$

$$x = x(s, t)$$

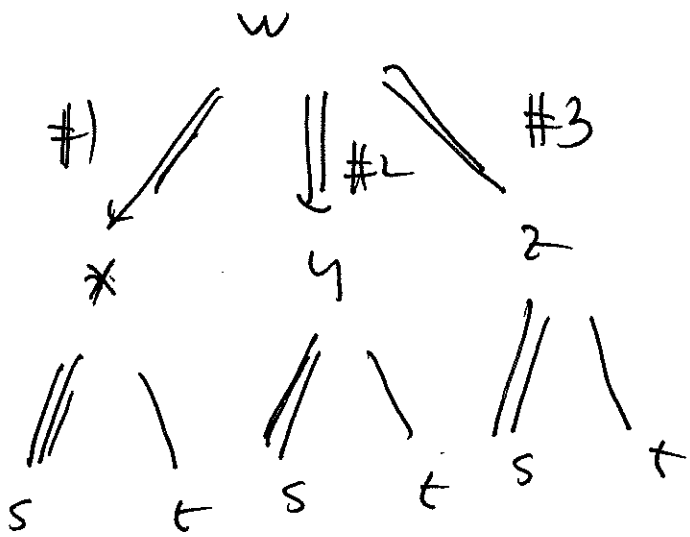
$$y = y(s, t)$$



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$w = f(x, y, t)$$
$$x = x(s, t)$$
$$y = y(s, t)$$

$$\frac{\partial w}{\partial s} =$$



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

$$\cos(xy) = 1 + \sin y$$

OR:

$$\cos(xy) - 1 - \sin y = 0$$

(y defined implicitly as a function of x)

$$F(x,y) = \cos(xy) - 1 - \sin y$$

$$\frac{\partial F}{\partial x} = -\sin(xy) \cdot y$$

$$\frac{\partial F}{\partial y} = -\sin(xy) \cdot x - \cos(y)$$

$$F(x,y) = 0$$

$$dF = 0 = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} \cdot dy$$

$$\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} \cdot dy = 0$$

$\div\, dx$:

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\boxed{\frac{dy}{dx}} = 0$$

$$\frac{\partial F}{\partial y}\boxed{\frac{dy}{dx}} = -\frac{\partial F}{\partial x}$$

$$\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y}$$