Math 213 - Lines and Planes (Part I of II)

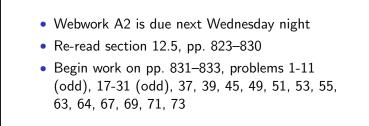
Peter A. Perry

University of Kentucky

January 18, 2019

Review

Homework



▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

Unit I: Geometry and Motion in Space

- Lecture 1 Three-Dimensional Coordinate Systems
- Lecture 2 Vectors
- Lecture 3 The Dot Product
- Lecture 4 The Cross Product
- Lecture 5 Equations of Lines and Planes, Part I
- Lecture 6 Equations of Lines and Planes, Part II
- Lecture 7 Cylinders and Quadric Surfaces
- Lecture 8 Vector Functions and Space Curves
- Lecture 9 Derivatives and integrals of Vector Functions
- Lecture 10 Motion in Space: Velocity and Acceleration
- Lecture 11 Functions of Several Variables

Lecture 12 Exam 1 Review

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

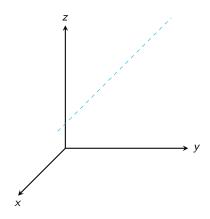
Review

Goals of the Day

- Learn how to write the parametric equation of a line
- · Learn how to write the symmetric equation of a line
- Learn how to write the vector equation of a plane
- Learn how to write the scalar equation of a plane

Review

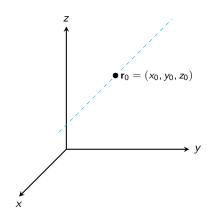
Line - Vector Equation



A line L in three-dimensional space is determined by

・ロト ・聞ト ・ヨト ・ヨト

Line - Vector Equation

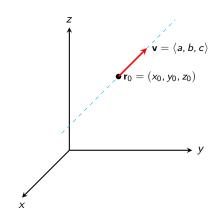


A line L in three-dimensional space is determined by

• A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line

(日)、

Line - Vector Equation



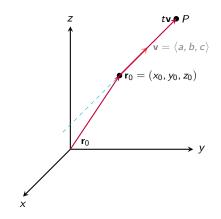
A line L in three-dimensional space is determined by

- A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line
- A vector v = (a, b, c) that gives the direction of the line

(日)、

Review

Line - Vector Equation



A line L in three-dimensional space is determined by

- A point $\mathbf{r}_0 = (x_0, y_0, z_0)$ on the line
- A vector v = (a, b, c) that gives the direction of the line

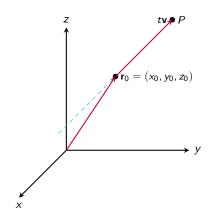
Any point P on the line can be expressed as



for some real number *t* called the *parameter*

イロト 不得 トイヨト イヨト

Line - Vector Equation



lf

$$\mathbf{r}_0 = \langle x_0, y_0, z_0
angle, \quad \mathbf{v} = \langle a, b, c
angle,$$

the function

 $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$

traces out a line through

 $P = (x_0, y_0, z_0)$

in the direction of

 $\mathbf{v}=\langle a,b,c
angle$

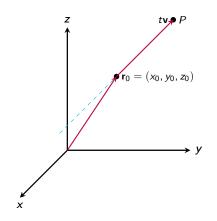
イロト 不得 トイヨト イヨト

Plane - Vector

Plane - Scalar

Review

Line - Parametric Equation



If
$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

then

 $x(t) = x_0 + at$ $y(t) = y_0 + bt$ $z(t) = z_0 + ct$

◆□> ◆□> ◆目> ◆目> ◆目 ● のへで

Line - Parametric Equation

If $\mathbf{r}(t) = \langle x(t), y(t), z(t)
angle$ then

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

gives the parametric equations for a line through $P(x_0, y_0, z_0)$ in direction $\langle a, b, c \rangle$

- 1. Find the parametric equations of a line L through the points P(1, 2, -1) and Q(2, 3, 4).
- 2. Find the parametric equations of the line L through the point (1, 2, 3)and parallel to the vector (2, -3, 4)

Plane - Vecto

Plane - Scalar

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

Review

Line - Symmetric Equation

If we begin with the parametric equations of a line:

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Review

Line - Symmetric Equation

If we begin with the parametric equations of a line:

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

we can eliminate the parameter to get the symmetric equation of a line;

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

The numbers (a, b, c) are the *direction numbers* of the line.

▲□▶ ▲□▶ ▲□▶ ▲□▶ = 三 のへで

Line - Symmetric Equation

If we begin with the parametric equations of a line:

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

we can eliminate the parameter to get the symmetric equation of a line;

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

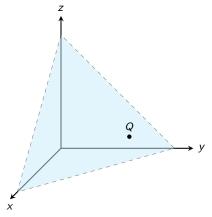
The numbers (a, b, c) are the *direction numbers* of the line.

- 1. Find the parametric and symmetric equations of the line through the origin and the point $({\rm 4,3,-1})$
- 2. Find the parametric and symmetric equations of the line through (2, 1, 0) and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

Plane - Vector

Review

Plane - Vector Equation

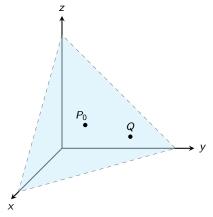


A *plane* is the collection of all points *Q*:

・ロト ・個ト ・モト ・モト

Review

Plane - Vector Equation



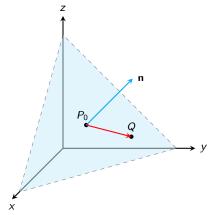
A *plane* is the collection of all points *Q*:

• Passing through given point $P_0(x_0, y_0, z_0)$

・ロト ・聞ト ・ヨト ・ヨト

Review

Plane - Vector Equation

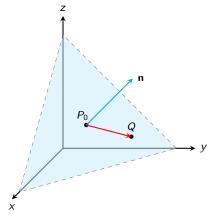


A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the normal vector

・ロト ・聞ト ・ヨト ・ヨト

Plane - Vector Equation



A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the normal vector

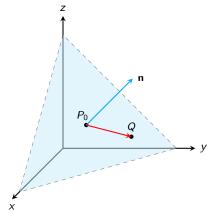
That is

$$\mathbf{n} \cdot \overrightarrow{P_0 Q} = \mathbf{0}$$

・ロト ・聞ト ・ヨト ・ヨト

Review

Plane - Vector Equation



A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the normal vector

That is

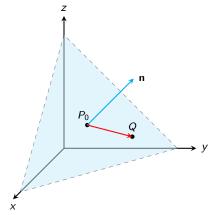
$$\mathbf{n}\cdot\overrightarrow{P_0Q}=\mathbf{0}$$

If
$$\mathbf{r}_0 = \overrightarrow{OP_0}$$
, $\mathbf{r} = \overrightarrow{OQ}$, then...

・ロト ・聞ト ・ヨト ・ヨト

Review

Plane - Vector Equation



A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the normal vector

That is

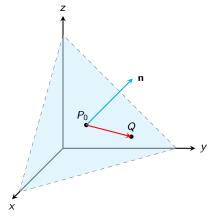
$$\mathbf{n}\cdot\overrightarrow{P_0Q}=\mathbf{0}$$

If
$$\mathbf{r}_0 = \overrightarrow{OP_0}$$
, $\mathbf{r} = \overrightarrow{OQ}$, then...

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = \mathbf{0}$$
 OR $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

◆□ > ◆□ > ◆豆 > ◆豆 > ̄豆 = のへで

Plane - Scalar Equation



A *plane* is the collection of all points *Q*:

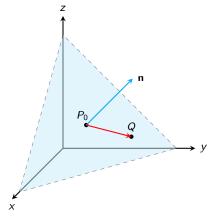
- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the *normal vector* That is

$$\mathbf{n} \cdot \overrightarrow{P_0 Q} = \mathbf{0}$$

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ ─臣 ─の�@

Review

Plane - Scalar Equation



A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the *normal vector* That is

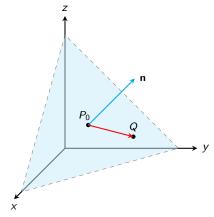
$$\mathbf{n} \cdot \overrightarrow{P_0 Q} = \mathbf{0}$$

If
$$Q = (x, y, z)$$
, $\mathbf{n} = \langle a, b, c \rangle$, then ...

(日)、

Review

Plane - Scalar Equation



A *plane* is the collection of all points *Q*:

- Passing through given point $P_0(x_0, y_0, z_0)$
- Having the property that $\overrightarrow{P_0Q}$ is perpendicular to a vector **n**, the *normal vector* That is

$$\mathbf{n} \cdot \overrightarrow{P_0 Q} = 0$$

If
$$Q = (x, y, z)$$
, $\mathbf{n} = \langle a, b, c \rangle$, then ...

(日) (同) (日) (日)

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Plane Puzzlers

Let

$$P_0 = P_0(x_0, y_0, z_0), \quad \mathbf{n} = \langle a, b, c \rangle, \quad P = P(x, y, z)$$

The **vector equation** of the plane through P_0 with normal **n** is

 $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

where **r** and \mathbf{r}_0 are position vectors for P and P_0 respectively. The **scalar equation** of the plane through P_0 with normal **n** is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

- 1. Find the vector equation of a plane through the origin and perpendicular to the vector $\langle -1,2,5\rangle$
- 2. Find the scalar equation of the plane through (1, -1, -1) and parallel to the plane 5x y z = 6
- 3. Find the equation of the plane that contains the line x = 1 + t, y = 2 t, z = 4 3t and is parallel to the plane 5x + 2y + z = 1

Review

Summary

- We learned that a line is determined by a point $P_0 = (x_0, y_0, z_0)$ on the line, and a vector $\mathbf{v} = \langle a, b, c \rangle$ that points along the line
 - The parametric equations of a line are

$$x(t) = x_0 + at$$
, $y(t) = y_0 + bt$, $z(t) = z_0 + ct$

The symmetric equations of a line are

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

- We learned that a plane is determined by a point $P_0 = (x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n} = \langle a, b, c \rangle$ normal to the plane
 - The *vector* equation of a plane is

$$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0)=\mathbf{0}$$

• The *scalar* equation of a plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$