Learning Goals

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Math 213 - Lines and Planes (Part II of II)

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University of Kentucky

January 23, 2019

Learning Goals

Visualizin

Distances

Summary

Homework

- Webwork A2 on 12.3 -12.4, the dot and cross products, is due tonight
- Webwork A3 on 12.5, equations of lines and planes, is due Friday
- Re-re-read section 12.5, pp. 823-830
- Finish work on pp. 831–833, problems 1-11 (odd), 17-31 (odd), 37, 39, 45, 49, 51, 53, 55, 63, 64, 67, 69, 71, 73
- Study for your quiz tomorrow on 12.4, the cross product
- For Friday: Review from last term: section 10.5 on **conic sections**
- For Friday: Read section 12.6, pp. 834-839

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Summary

Unit I: Geometry and Motion in Space

- Lecture 1 Three-Dimensional Coordinate Systems
- Lecture 2 Vectors
- Lecture 3 The Dot Product
- Lecture 4 The Cross Product
- Lecture 5 Equations of Lines and Planes, Part I
- Lecture 6 Equations of Lines and Planes, Part II
- Lecture 7 Cylinders and Quadric Surfaces
- Lecture 8 Vector Functions and Space Curves
- Lecture 9 Derivatives and integrals of Vector Functions
- Lecture 10 Motion in Space: Velocity and Acceleration
- Lecture 11 Functions of Several Variables

Lecture 12 Exam 1 Review

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Summary

Goals of the Day

- Review dot, cross, and triple scalar products
- Review equations of lines and planes
- Sketch and visualize lines and planes
- Learn how find the distance from a point to a plane

Summary

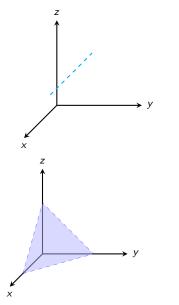
Dot Product, Cross Product, Triple Product

	Formula	Туре	Geometry	Zero if
Dot	a · b	Scalar	Projections	a, b orthogonal
Cross	$\mathbf{a} \times \mathbf{b}$	Vector	Area of a Parallelogram	a, b parallel
Triple	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	Scalar	Volume of a Parallelepiped	a, b, c coplanar

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Summary

Lines and Planes



To specify the equation of a **line** L, you need:

To specify the equation of a **plane**, you need:

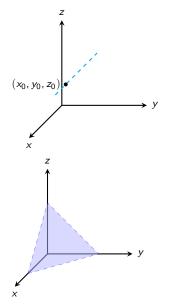
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Summary

Lines and Planes



To specify the equation of a **line** L, you need:

• A point (x_0, y_0, z_0) on L

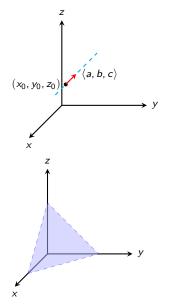
To specify the equation of a **plane**, you need:

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Summary

Lines and Planes



To specify the equation of a line L, you need:

- A point (x_0, y_0, z_0) on L
- A vector (a, b, c) in the direction of L

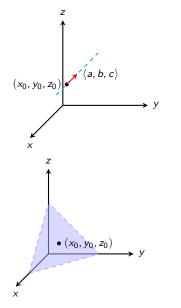
To specify the equation of a **plane**, you need:

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Summary

Lines and Planes



To specify the equation of a line L, you need:

- A point (x_0, y_0, z_0) on L
- A vector (a, b, c) in the direction of L

To specify the equation of a **plane**, you need:

• A point (x₀, y₀, z₀) on the plane

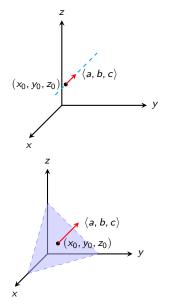
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Summary

Lines and Planes



To specify the equation of a line L, you need:

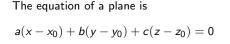
- A point (x_0, y_0, z_0) on L
- A vector (a, b, c) in the direction of L

To specify the equation of a **plane**, you need:

- A point (x₀, y₀, z₀) on the plane
- A vector n = (a, b, c) normal to the plane

Summary

Hot Tip - Planes Made Simple



or

$$ax + by + cz = d$$

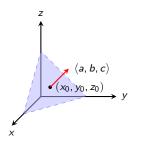
- Step 1. Determine $\langle a, b, c \rangle$ from geometry
- Step 2. Find *d* by substituting in x_0 , y_0 , z_0

Example: Find the equation of a plane parallel to the plane

$$x - y + 2z = 0$$

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through the point (2, 2, 2).



Summary

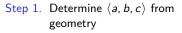
Hot Tip - Planes Made Simple

The equation of a plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

or

$$ax + by + cz = d$$



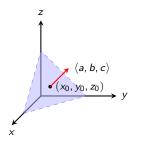
Step 2. Find *d* by substituting in x_0 , y_0 , z_0

Example: Find the equation of a plane orthogonal to the line

$$(x, y, z) = (-7, 0, 0) + t(-7, 3, 3)$$

which passes through the point (0, 0, -7). Give your answer in the form ax + by + cz = d where a = 7.

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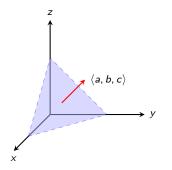
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Summary

Hot Tip - Sketching Planes Made Simple

The equation of a plane is

ax + by + cz = d



Hot Tip - Sketching Planes Made Simple

The equation of a plane is

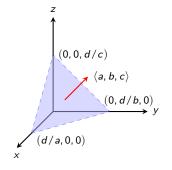
$$ax + by + cz = d$$

To sketch the plane with this equation, you can find the x-, y-, and z-intercepts from the equation:

$$x = d/a$$
, $y = d/b$, $z = d/c$

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Hot Tip - Sketching Planes Made Simple

The equation of a plane is

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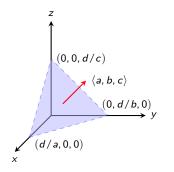
Sketch the part of the plane

$$2x + y + 3z = 4$$

in the first octant and label the x- , y-, and z-intercepts.

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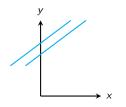
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Intersecting, Parallel, and Skew Lines

In two-dimensional space, two lines $L_1 \mbox{ and } L_2 \mbox{ can be }$



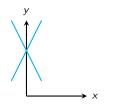
In two-dimensional space, two lines L_1 and L_2 can be

• parallel, or

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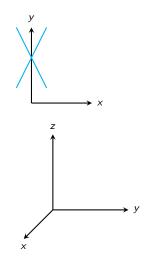
Summary

Intersecting, Parallel, and Skew Lines



In two-dimensional space, two lines $L_1 \mbox{ and } L_2 \mbox{ can be }$

- parallel, or
- intersecting



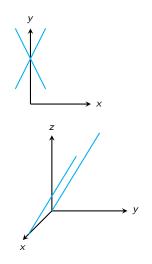
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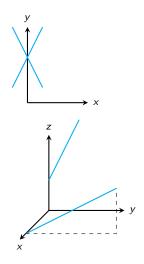
- parallel, or
- intersecting

In three dimensions, two lines L_1 and L_2 can be

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• parallel,



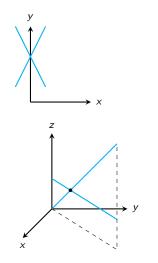
In two-dimensional space, two lines $L_1 \mbox{ and } L_2 \mbox{ can be }$

- parallel, or
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In three dimensions, two lines L_1 and L_2 can be

- parallel,
- skew, or





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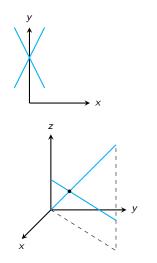
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In three dimensions, two lines L_1 and L_2 can be

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- parallel,
- skew, or
- intersecting



In two-dimensional space, two lines $L_1 \mbox{ and } L_2 \mbox{ can be }$

- parallel, or
- intersecting

In three dimensions, two lines L_1 and L_2 can be

- parallel,
- skew, or
- intersecting

How do you tell which is which?

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Summary

Intersecting, Parallel, and Skew Lines

 $\mathbf{r}(t) = \mathbf{r}_0(t) + t\mathbf{v}$

- Two lines are parallel if the corresponding vectors ${\bf v}$ are parallel
- If not parallel, two lines intersect if we can solve for the point of intersection
- If not parallel, and nonintersecting, they are skew

Determine whether the following pairs of lines are parallel, intersect, or are skew. If they intersect, find the points of intersection.

1.
$$L_1: x = 2 + s$$
, $y = 3 - 2s$, $z = 1 - 3s$
 $L_2: x = 3 + t$, $y = -4 + 3t$, $z = 2 - 7t$
2. $L_1: \frac{x}{1} = \frac{y - 1}{-1} = \frac{z - 1}{-3}$, $L_2: \frac{x - 2}{2} = \frac{y - 3}{-2} = \frac{z}{7}$

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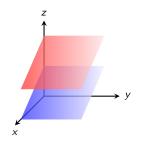
Intersecting and Parallel Planes

Two planes either

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Summary

Intersecting and Parallel Planes



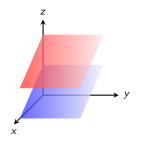
Two planes either

• are parallel (if their normal vectors are parallel), or

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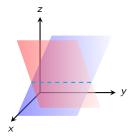
Summary

Intersecting and Parallel Planes



Two planes either

- are parallel (if their normal vectors are parallel), or
- intersect in a line

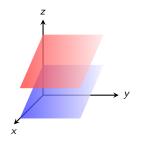


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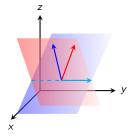
Intersecting and Parallel Planes



Two planes either

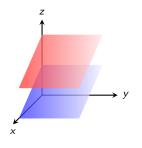
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- intersect in a line

A vector pointing along that line will be perpendicular to *both* normal vectors



Summary

Intersecting and Parallel Planes



x x Two planes either

- are parallel (if their normal vectors are parallel), or
- intersect in a line

A vector pointing along that line will be perpendicular to *both* normal vectors

Find the line of intersection between the planes

$$x + 2y + 3z = 1$$

and

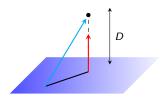
$$x - y + z = 1$$

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Summary

The Distance from a Point to a Plane

To find the distance \boldsymbol{D}

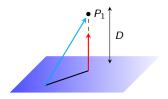


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Summary

The Distance from a Point to a Plane

To find the distance D from a point P_1

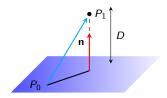


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Summary

The Distance from a Point to a Plane

To find the distance *D* from a point P_1 to a plane with normal vector **n** containing a point P_0 :



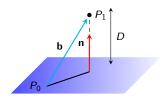
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Summary

The Distance from a Point to a Plane

To find the distance *D* from a point P_1 to a plane with normal vector **n** containing a point P_0 :

Let **b** be the vector $\overrightarrow{P_0P_1}$



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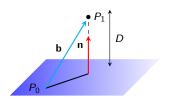
The Distance from a Point to a Plane

To find the distance *D* from a point P_1 to a plane with normal vector **n** containing a point P_0 :

Let **b** be the vector $\overrightarrow{P_0P_1}$

Then the distance D is given by $\operatorname{comp}_{\mathbf{n}} \mathbf{b}$, or

$$D = \frac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|}$$



The Distance from a Point to a Plane

To find the distance *D* from a point P_1 to a plane with normal vector **n** containing a point P_0 :

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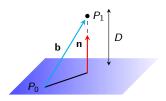
$$P_1 = P_1(x_1, y_1, z_1),$$

$$P_0 = P_0(x_0, y_0, z_0),$$

then

$$\mathbf{b} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

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Summary

The Distance from a Point to a Plane

$$D = \frac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|}$$

$$\mathbf{b} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\mathbf{n} = \langle a, b, c \rangle$$

If the plane's equation is

$$ax + by + cz + d = 0$$

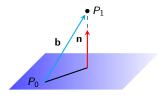
then

$$\mathbf{n} \cdot \mathbf{b} = a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)$$

= $ax_1 + by_1 + cz_1 + d$

so

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$





- We reviewed the basic facts about the dot product $\mathbf{a} \cdot \mathbf{b}$, the cross product $\mathbf{a} \times \mathbf{b}$, and the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
- We reviewed how to write equations of lines and planes and ...
 - How to determine whether two lines are parallel, perpendicular, or skew

- · How to determine whether planes are parallel or intersecting
- We computed the distance from a point to a plane