Math 213 Exam 1

Name: ______________________ Section: __________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used other than a one-page "cheat sheet" of notes, formulas, etc., written or typeset on one or both sides of an 8-1/2″ × 11″ sheet of paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 6 free-response questions. Please follow these guideines to receive maximum credit.

- Each question is followed by a space to write your answer. Please write your answer *neatly* in the space provided.
- Show all work to receive full credit on the free response problems. You will be graded on the clarity of your presentation as well as the correctness of your answers.
- Give exact answers, rather than decimal equivalents, unless otherwise instructed (e.g., $\sqrt{2}$, not 1.414).

Question	1	2	3	4	5	6	**Total**
Possible	10	18	18	18	18	18	100
Score							

1. (Vector Basics - 10 points) Suppose that $\mathbf{a} = \langle 3, 4 \rangle$, $\mathbf{b} = \langle 9, -1 \rangle$. Find:

   (a) (3 points) $4\mathbf{a} + 2\mathbf{b}$

   (b) (4 points) $|\mathbf{a} - \mathbf{b}|$

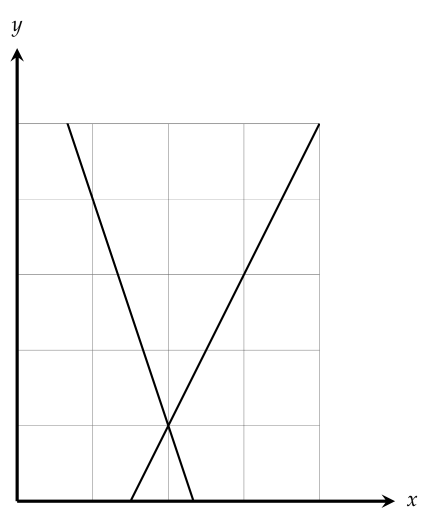
   (c) (3 points) $\mathbf{a} \cdot \mathbf{b}$

2. (Dot and Cross Products - 18 points)

(a) (4 points) If $\mathbf{a} = \langle 2, -1, 3\rangle$ and $\mathbf{b} = \langle 4, 2, 1\rangle$, find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

(b) (4 points) Use the scalar triple product to verify that the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar. Be sure to justify your conclusion.

(c) (10 points) Using dot products, find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$ shown below. Be sure to explain how the dot product is used, with what vectors, and why!



3. (18 points - Equations of Lines and Planes)

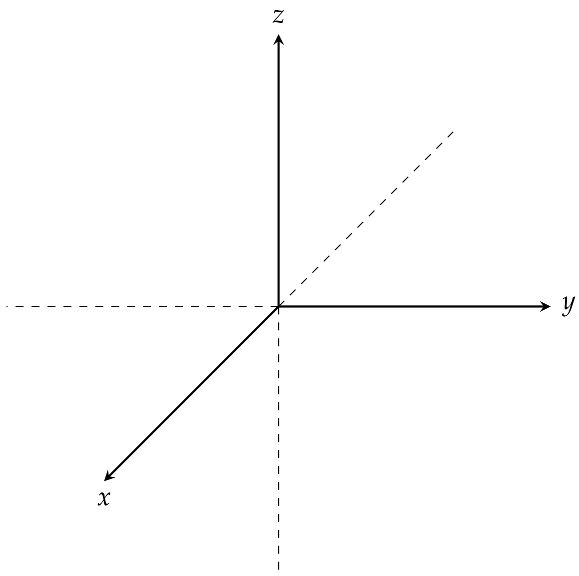
   (a) (6 points) Find the parametric and symmetric equations for the line through the points $(0, 0, 0)$ and $(4, 3, 1)$.

   (b) (6 points) Find the equation of the plane through the origin and perpendicular to the vector $\langle 1, -2, 4 \rangle$.

   (c) (6 points) Determine whether the planes $x + 2y - z = 2$ and $2x - 2y + z = 1$ are parallel, perpendicular, or neither.

4. (Quadric Surfaces - 18 points)

   (a) (9 points) Sketch the surface $x^2 + z^2 = 1$ on the axes provided. Also, please provide the information requested below the sketch.

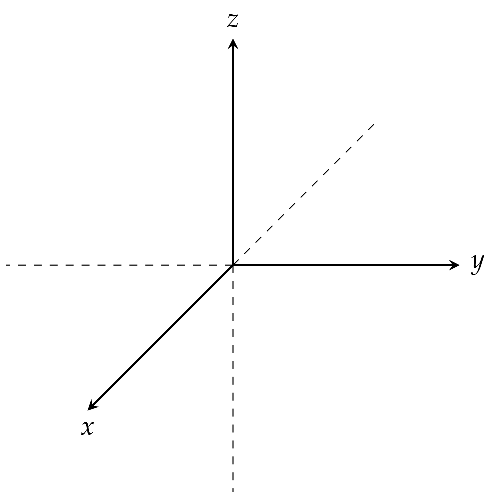


Specify the axis of symmetry:

Describe the cross-sections perpendicular to the axis of symmetry.

Name the surface:

(b) (9 points) Sketch the surface $x^2 + z^2 = y^2$ on the axes provided. Also, please provide the information requested below your sketch.



Identify the traces in planes parallel to the xy-plane by equation and by name.

Identify the traces in planes parallel to the xz-plane by equation and by name.

Identify the traces in planes parallel to the yz-plane by equation and by name.

Name the surface:

5. (Vector Functions, Derivatives, Integrals - 18 points)

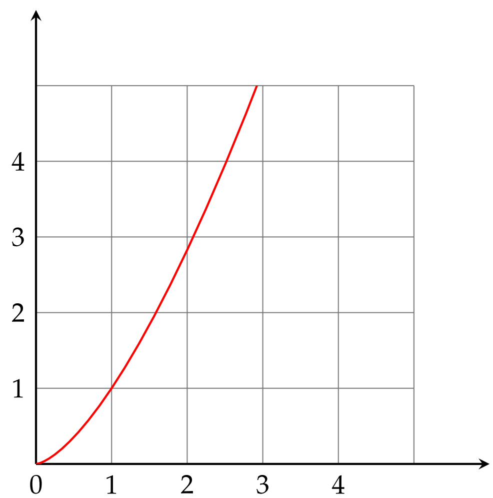
   (a) (4 points) Find the unit tangent $\mathbf{T}(t)$ to the curve

   $$\mathbf{r}(t) = \langle t^2 - 2t, 1 + 3t, t^3/3 + t^2/2 \rangle$$

   at $t = 2$.

   (b) (3 points) Find a formula for $\mathbf{r}'(t)$ for the vector function $\mathbf{r}(t) = \langle t^2, t^3 \rangle$.

   (c) (4 points) Below is the curve $\mathbf{r}(t)$ from part (b) for $0 \le t \le \sqrt{3}$. Sketch and label the position vector $\mathbf{r}(1)$ starting at the origin and label the point P having position vector $\mathbf{r}(1)$. Then sketch and label the tangent vector $\mathbf{r}'(1)$ starting at the point P.



(d) (7 points) Find a vector function that represents the intersection of the hyperbolic paraboloid ("saddle") $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$. *Hint*: First parameterize x and y to move along the circle of radius 1.

6. (Motion in Space - 18 points)

   A projectile is fired with initial speed of 200 m/s and angle of elevation 60°. Recall that the acceleration due to gravity is $9.8\ \text{m/sec}^2$.

   (a) (3 points) Find the initial velocity vector $\mathbf{v}_0$.

   (b) (3 points) Find the equation for $\mathbf{r}(t)$, the position of the projectile as a function of time.

   (c) (3 points) Find the time of impact.

   (d) (3 points) Find the maximum height reached.

   (e) (3 points) Find an expression for $\mathbf{r}'(t)$, the velocity of the projectile.

   (f) (3 points) Find the speed of the projectile at impact.