Quiz 10

Name: ____________________ Section and/or TA: ____________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Find the curl and the divergence of the vector field

$$\mathbf{F}(x,y,z) = \sin(yz)\mathbf{i} + \sin(xz)\mathbf{j} + \sin(xy)\mathbf{k}.$$

Solution: For divergence, we have

$$\text{div}\,\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}\sin(yz) + \frac{\partial}{\partial y}\sin(xz) + \frac{\partial}{\partial z}\sin(xy) = 0.$$

For curl, we have

$$\text{curl}\,\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin(yz) & \sin(xz) & \sin(xy) \end{vmatrix}$$

$$= \big(x\cos(xy) - x\cos(xz)\big)\mathbf{i} - \big(y\cos(xy) - y\cos(yz)\big)\mathbf{j} + \big(z\cos(xz) - z\cos(yz)\big)\mathbf{k}$$

2. (3 points) Find an equation of the tangent plan to the surface with parametric equations $x = u^2 + 2v^3$, $y = 3u$, and $z = 5v^2$ at the point $(2,6,5)$.

Solution: First we find

$$\mathbf{r}_u = 2u\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{r}_v = 6v^2\mathbf{i} + 10v\mathbf{k}.$$

Then

$$\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2u & 3 & 0 \\ 6v^2 & 0 & 10v \end{vmatrix}$$

$$= 30v\mathbf{i} - 20uv\mathbf{j} - 18v^2\mathbf{k}$$

Since $(x,y,z) = (2,6,5)$ give $u = 2$ and $v = -1$, we have

$$\mathbf{r}_u \times \mathbf{r}_v = -30\mathbf{i} + 40\mathbf{j} - 18\mathbf{k}.$$

So an equation for the tangent line is

$$-30(x-2)+40(y-6)-18(z-5)=0$$

or

$$-30x+40y-18z=90.$$