Math 575

Exam 1 Review

Your first exam will cover the sections we've covered from chapters 1–4 from Beals' book *Analysis: An Introduction*. What follows is a list of essential definitions and theorems. In your studying you should be sure to know these definitions, the theorems, and their proofs. You should also review homework problems and try other problems in Beals' book from the sections covered.

Your exam will consists of four problems worth 25 points each. Each problem will include a proof but you also may be asked to state a relevant definition or to give a counterexample.

Definitions to Know

- Least upper bound of a set of real numbers
- Greatest lower bound of a set of real numbers
- Limit of a sequence of complex numbers
- Monotone sequence of real numbers (nonincreasing or nondecreasing)
- $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$ for a sequence $\{x_n\}$ of real numbers
- Cauchy sequence (for Cauchy criterion)
- Subsequence
- Extended real numbers
- Convergence of series
- Absolute convergence of series
- Conditional convergence of series
- Power series
- Radius of convergence of a power series

Theorems to Know

Here "know" means not only know the statement but also understand the proof!

- Every bounded monotone sequence converges to a limit
- Limit arithmetic (the limit of a sum is the sum of limits, etc.)
- Every Cauchy sequence of complex numbers converges to a limit
- Every bounded sequence has a convergent subsequence (Bolzano-Weierstrass Theorem)
- Every absolutely convergent sequence is convergent
- Comparison, Ratio and Root Tests
- A power series $\sum_{n=0}^{\infty} a_n z^n$ either converges for all complex numbers z, or for all z with $|z| < R$, or converges only at $z = 0$
- Suppose that $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence $R > 0$. The function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is differentiable in the complex sense and

  $$f'(z) = \sum_{n=1}^{\infty} n a_n z^{n-1}$$

  where the series converges for $|z| < R$.