Calculus I
Examination 1

6 February 2007

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please: 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*, 3) give exact answers, rather than decimal approximations to the answer.
Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question. If you use the back of a sheet, please indicate this by the question.
You are to answer two of the last three questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name Solutions

Section ____________

Last four digits of student identification number ____________

Question	Score	Total
p. 1/Q1–2		14
p. 2/Q3–4		14
p. 3/Q5		14
p. 4/Q6–7		14
p. 5/Q8		14
p. 6/Q9		14
p. 7/Q10		14
p. 8/Q11		14
Free	2	2
		100

1. Find the equation of the line that passes through $(1, 2)$ and is parallel to the line $4x + 2y = 11$. Put your answer in the form $y = mx + b$.

$2y = 11 - 4x$

$y = -2x + \frac{11}{2}$

Slope is -2

(2) slope

Line w/ slope -2 and passing through $(1,2)$ is

$y - 2 = -2(x-1)$. (3)

$y = -2x + 2 + 2$

$= -2x + 4$. (2) simplify

$y = -2x + 4$.

2. An object moves so that at time t seconds it is located $s(t) = t^3 + 2t$ meters to the right of a reference point.

(a) Find the average velocity of the object for the interval $1 \leq t \leq 3$.

(b) Does the object move to the left or the right during the interval $1 \leq t \leq 3$?

a) $V_{avg} = \frac{S(3) - S(1)}{3 - 1} = \frac{27 + 6 - 3}{2}$ (3) formula

$= \frac{30}{2} = 15$ (1) Number

(1) units

(b) $V_{avg} > 0$ or $S(3) > S(1)$.

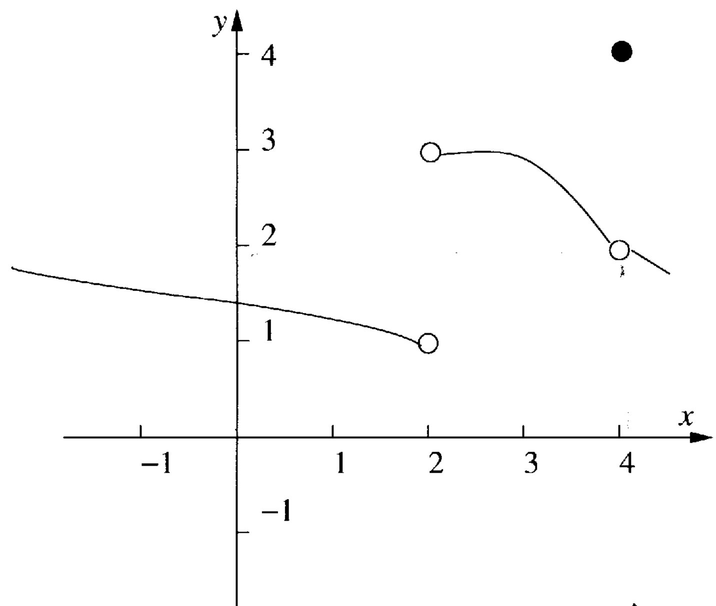
So movement is to right

(2) Answer only

(a) 15 meter/second, (b) Right

3. Let f be the function whose graph is below. For each limit, give the value or explain why the limit does not exist.

(a) $\lim_{x\to 2} f(x)$ (b) $\lim_{x\to 4} f(x)$.



(a) Limit doesn't exist since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$. ① Reaso

(a) $\lim_{x\to 2} f(x) =$ DNE ②, (b) $\lim_{x\to 4} f(x) =$ 2 ④

4. Let $f(x) = \dfrac{1}{\sqrt{2-x}}$ and $g(x) = x^2$.

(a) Find the composite function h defined by $h(x) = f(g(x))$.

(b) Find all x so that the function h is continuous at x.

$h(x) = \dfrac{1}{\sqrt{2-x^2}}$

(a) $h(x) = \dfrac{1}{\sqrt{2-x^2}}$ ③

(b) $\{x : -\sqrt{2} < x < \sqrt{2}\}$ ③

or $(-\sqrt{2}, \sqrt{2})$ ① correct notation.

5. For each limit, find the value or state that the limit does not exist. Explain your reasoning.

(a) $\lim_{x\to 2} \dfrac{x^2-3x+2}{x^2+2x-8}$

(b) $\lim_{x\to 2} \dfrac{x^2+2x+1}{x^2-4}$

(c) $\lim_{x\to 2} \dfrac{x^2-2x-3}{x^2+x-2}$

(a) $\dfrac{x^2-3x+2}{x^2+2x-8} = \dfrac{(x-1)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$

$\lim_{x\to 2} \dfrac{x-1}{x+4} = \dfrac{1}{6}$.

3 Simplify

2 Answer

(b) $\dfrac{(x^2+2x+1)}{(x^2-4)} = \dfrac{(x+1)^2}{(x-2)(x+2)}$

1 Reason

$\lim_{x\to 2^+} \dfrac{(x+1)^2}{(x-2)(x+2)} = +\infty$. $\lim_{x\to 2^-} \dfrac{(x+1)^2}{(x-2)(x+2)} = -\infty$.

3

So limit does not exist.

c) $\dfrac{(x-3)(x+1)}{(x+2)(x-1)}$ $\lim_{x\to 2} \dfrac{x^2-2x-3}{(x+2)(x-1)}$

$= \dfrac{4-7}{4} = -3/4$.

5

Use rule for limit of quotients, since $\lim_{x\to 2} (x+2)(x-1) \neq 0$.

(a) 1/6, (b) Doesn't exist, (c) -3/4.

6. Suppose that $\lim_{x\to 2}(x^2 f(x) + 2x) = 5$. Find $\lim_{x\to 2} f(x)$.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\left(\frac{x^2 f(x) + 2x}{x^2}\right) - \lim_{x\to 2}\frac{2x}{x^2}.$$ 4

$$= \frac{\lim_{x\to 2}(x^2 f(x) + 2x)}{\lim_{x\to 2} x^2} - \lim_{x\to 2}\frac{2}{x} = \frac{5}{4} - 1 = 1/4.$$ 3

Or.

$$5 = \lim_{x\to 2}(x^2 f(x) + 2x) = \lim_{x\to 2} x^2 \lim_{x\to 2} f(x) + \lim_{x\to 2} 2x$$ 4

$$= 4 \lim f(x) + 4.$$

So $5 - 4 = 4 \lim_{x\to 2} f(x)$

$1/4 = \lim_{x\to 2} f(x)$ 3

$\lim_{x\to 2} f(x) =$ 1/4.

7. Let $f(x) = x^5 + 2x + 2$. Find an interval $[a, b]$ so that equation $f(x) = 0$ has a solution in the interval $[a, b]$. Use the intermediate value theorem to explain why there is a solution in the interval you found.

$f(x)$ is polynomial and hence continuous.

$f(-1) = -1 - 2 + 2 = -1.$

$f(0) = 2$

$f(-1) < 0$, $f(0) = 2 > 0$. 3

Hence there is a c in $[-1, 0]$ so that $f(c) = 0$

There is a solution in the interval $[-1, 0]$ or $(-1, 0)$. 4

Many correct solutions.

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

9. (a) State the principle of mathematical induction. Use complete sentences.

   (b) Find

$$\sum_{k=1}^{1}(2k+1) \qquad \text{and} \qquad \sum_{k=1}^{2}(2k+1).$$

   (c) Use the principle of mathematical induction to prove that for $n = 1, 2, \ldots$ we have

$$\sum_{k=1}^{n}(2k+1) = n^2 + 2n.$$

4 (a) If $P_1, P_2, P_3 \ldots$ are statements and suppose

1 (1) P_1 is true. (2) If P_N is true, then P_{N+1} is true.

Sen. If (1) and (2) hold, then all the statements $P_1, P_2, P_3 \ldots$ are true.

2 (b) $\sum_{k=1}^{1} 2k+1 = 3.$ $\quad \sum_{k=1}^{2} 2k+1 = 3+5 = \underline{8}$

2 (c) (Base) If $n=1$, $n^2+2n = 3$ and $\sum_{k=1}^{n} 2k+1 = 3.$

5 (Induction step): Assume $\sum_{k=1}^{N} 2k+1 = N^2+2N.$

Add $2(N+1)+1 = 2N+3$ to both sides.

Then $\sum_{k=1}^{N+1} 2k+1 = 2(N+1)+1 + \sum_{k=1}^{N} 2k+1. = N^2+2N+2(N+1)+1.$

$= (N+1)^2 + 2(N+1).$

Thus, the principle of mathematical induction implies $\sum_{k=1}^{n} 2k+1 = n^2+2n$ for $n = 1, 2, 3 \ldots$

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2 (c) (Base) to $n=1$, $n^2+2n = 3$ and $\sum_{k=1}^{n} 2k+1 = 3$.

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$= (N+1)^2 + 2(N+1)$.

Thus, the principle of mathematical induction implies $\sum_{k=1}^{n} 2k+1 = n^2+2n$ for $n = 1, 2, 3 \ldots$

(a) State the definition of the derivative of a function f at a number a. Use complete sentences.

(b) Let

$$g(x) = \frac{1}{3x-2}.$$

Use the definition of the derivative to find the derivative g'.

(c) Give the domain of the derivative g'.

(a) The derivative of f at a is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided the limit exists

③

① Sentence

(b) $\frac{g(x+h) - g(x)}{h} = \left(\frac{1}{3(x+h)-2} - \frac{1}{3x-2}\right) \frac{1}{h}.$ ②

$$= \frac{(3x-2) - (3x+3h-2)}{(3(x+h)-2)(3x-2)} \cdot \frac{1}{h}$$

③ Simplify

$$= \frac{-3\cancel{h}}{\cancel{h}} \cdot \frac{1}{(3(x+h)-2)(3x-2)}.$$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{-3}{(3(x+h)-2)(3x-2)}$$

$$= \frac{-3}{(3x-2)^2}$$

③ Limit

(c) g' is defined if $3x - 2 \neq 0$ or $x \neq 2/3$. The domain is $\{x : x \neq 2/3\}$.

①

① Correct notation.

11. (a) Define what it means for a function f to be continuous at a number a. Use complete sentences.

(b) Let b and c be numbers and define a function g by

$$g(x) = \begin{cases} 2-x, & x < 1 \\ bx + c, & 1 \le x \le 2 \\ x^2, & 2 < x \end{cases}$$

Find

$\lim_{x\to 1^+} g(x) =$ $b+c$ $\qquad \lim_{x\to 1^-} g(x) =$ 1

$\lim_{x\to 2^+} g(x) =$ 4 $\qquad \lim_{x\to 2^-} g(x) =$ $2b+c$.

Some of your answers will involve b and c.

(c) Find values for b and c so that the function g in part (b) is continuous for all real numbers.

a) A function f is continuous at a if $\lim_{x\to a} f(x) = f(a)$. 3 1 sentence

b) 4 points, 1 each.

c) Need $b+c=1$.

$2b+c=4$ 3

$b=3$

$c=1-3=-2$ 3

$\underline{b=3}$, $\underline{c=-2}$

Check $b+c = 3+-2 = \underline{1}$

$2b+c = 6+-2 = \underline{4}$