Worksheet 6. MA113
Calculus I Fall 2006

As always, your work should be written out neatly and carefully. Use complete sentences.
Consider the equation $x^3 + y^3 = 3xy$ and consider the curve formed by all points (x, y) in the first quadrant which satisfy this equation.

1. Find the point (x, y) in the first quadrant that lies on the curve and has the largest possible y-coordinate. To do this, use implicit differentiation to find dy/dx. Find the point where the curve has a horizontal tangent. Use implicit differentiation again to find d^2y/dx^2 and find the value of the second derivative at the critical point. Use the second derivative test to verify that the critical point is a local maximum for y. This point will have the largest y-coordinate.

2. Verify that the curve is symmetric with respect to the line $y = x$. This means that if a point (x, y) satisfies the equation, then the point (y, x) also satisfies the equation. Find the point in the first quadrant that lies on the graph of the given equation and has the largest possible x-coordinate. Hint: Since we know the point with the largest y-coordinate, this should be easy.

3. Find a parametric form for the curve as follows. By a parametric form for the curve, we mean two functions f and g so that each point on the curve (x, y) is given as $(x, y) = (f(m), g(m))$ for some m. To find the functions f and g, find the point of intersection of the line $y = mx$ with the graph of the equation above (there will be a single point for each positive m). Write the x and y-coordinates of the point of intersection as functions of m.

4. Sketch the graph of the equation by carefully plotting points. (Do this by choosing non-negative values for m, and then using the functions from part (3) to determine the point (x, y) for that m. Remember the curve is symmetric with respect to the line $y = x$, so for each point you find below the line $y = x$, you will be able to plot another point above the line $y = x$. You may check yourself by using dy/dx from part (1) and checking that the slopes are reasonable.)

5. (Extra Credit) What is the name of the curve we studied in this worksheet?

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