Worksheet 3.	MA113
Calculus I	Fall 2006

As always, your work should be written out neatly and carefully. Use complete sentences.

1. Consider a triangle with vertices A, B and C. The point D is on the side BC and the line segments AD and BC are perpendicular. Apply the Pythagorean theorem and the definition of the cosine function to show that if a, b and c are the lengths of the sides of the triangle and  $\gamma$  is the measure of the angle opposite the side of length c, then



$$c^2 = a^2 + b^2 - 2ab\cos\gamma.$$

2. Use the principle of mathematical induction to prove the differentiation rule for powers,

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad n = 1, 2, 3, \dots$$

Hint: The base case follows easily from the definition. For the induction step, write  $x^{N+1} = x \cdot x^N$  and use the product rule.

3. Use the definition of the derivative to prove that if f is differentiable at a number a and  $f(a) \neq 0$ , then the reciprocal g defined by g(x) = 1/f(x) is differentiable at a and

$$g'(a) = \frac{-f'(a)}{f(a)^2}.$$

In your paper, you should explain why f is continuous at a and why this is needed to find the derivative of g at a.

September 14, 2006