As always, your work should be written out neatly and carefully. Use complete sentences.

1. Consider a triangle with vertices $A, B$ and $C$. The point $D$ is on the side $B C$ and the line segments $A D$ and $B C$ are perpendicular. Apply the Pythagorean theorem and the definition of the cosine function to show that if $a, b$ and $c$ are the lengths of the sides of the triangle and $\gamma$ is the measure of the angle opposite the side of length $c$, then

$$
c^{2}=a^{2}+b^{2}-2 a b \cos \gamma
$$


2. Use the principle of mathematical induction to prove the differentiation rule for powers,

$$
\frac{d}{d x} x^{n}=n x^{n-1}, \quad n=1,2,3, \ldots
$$

Hint: The base case follows easily from the definition. For the induction step, write $x^{N+1}=x \cdot x^{N}$ and use the product rule.
3. Use the definition of the derivative to prove that if $f$ is differentiable at a number $a$ and $f(a) \neq 0$, then the reciprocal $g$ defined by $g(x)=1 / f(x)$ is differentiable at $a$ and

$$
g^{\prime}(a)=\frac{-f^{\prime}(a)}{f(a)^{2}} .
$$

In your paper, you should explain why $f$ is continuous at $a$ and why this is needed to find the derivative of $g$ at $a$.

September 14, 2006

