Worksheet 5. Rainbows. MA113
Calculus I Fall 2006

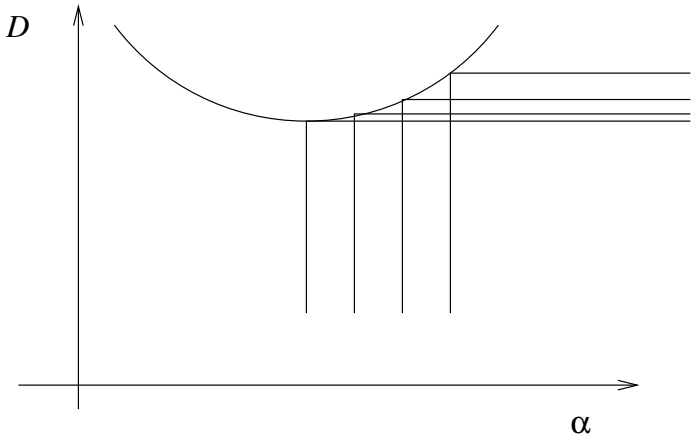
Write out your answers carefully and in complete sentences.

1. Let f be defined for all real numbers and suppose that $0 \leq f'(x) \leq 1$ for all x. Can we have $f(1) = 2$ and $f(4) = 6$? Find an example to show this is possible or give a careful explanation as to why it is impossible.

2. Carry out parts 1 and 2 of the project "The calculus of rainbows", page 232 of the fifth edition of Stewart. In part 1, you do not need to show that the critical number is a minimum. As we try to explain below, any critical number should lead to a concentration of light.

3. (1 point extra credit) The sky below a rainbow is often either brighter or darker than the sky above the rainbow. Is it brighter or darker? The critical number you found in part 1) is a minimum. Use that the critical number is a minimum and a sketch showing a few typical light rays passing through a raindrop to help explain whether it is brighter above or below the rainbow.

We have used critical numbers to help us find local extreme values for a function. This project shows another reason why the critical numbers of a function are important. If $f'(c) = 0$, then the linear approximation to f at c is a constant function (another name for a point where the derivative is zero is stationary point). When we see a rainbow in the sky, the rainbow is formed by light rays being concentrated near a critical point of the function $D(\alpha)$ discussed in this project. The drawing below helps to show why a rainbow corresponds to a critical number of $D(\alpha)$. The graph shows that near a critical number of D, equally spaced values of α, lead to values of D which are concentrated near the value of D at the critical point.



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