Worksheet 4. MA113
Calculus I Fall 2006

The following exercises provide some practice with the derivative.
As always, your work should be written out neatly and carefully. Use complete sentences.

1. (a) If $n = 1, 2, 3, \ldots$ is a natural number, prove that

$$\frac{d}{dx} \sin^n(x)\, \cos(nx) = n \sin^{n-1}(x)\, \cos((n+1)x).$$

   Hint: Do not use the principle of mathematical induction. Do remember the addition formula for the cosine function.

   (b) State and prove a formula for

$$\frac{d}{dx} \cos^n(x)\, \sin(nx)$$

   that is similar to the formula in part a).

2. A trough is 10 meters long and has a cross-section in the form of an equilateral triangle with sides of length 3 meters.

   (a) If the water in the tank is $x$ meters deep, find the volume of water in the tank as a function of $x$.

   (b) If the tank is being filled with water at the rate of 10 cubic meters per hour, how fast is the water level rising when the depth is 1 meter?

3. Minutes after a bank robbery, a police helicopter, hovering directly over the bank at an elevation of 600 meters discovers that the getaway car is speeding along a straight road leading directly away from the bank. At that instant, top secret advanced stealth sensing technology on the helicopter shows that the car is 1 kilometer away from the helicopter and is moving at a speed of 120 kilometers/hour.

   (a) Draw a picture showing the above information.

   (b) Find the ground speed of the car at the instant that the car is 1 kilometer from the helicopter.

4. (Extra credit) The father of the Professor's son is talking with the son of the Professor's father. Is the Professor necessarily talking?

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