Before beginning, it might be helpful to recall the quadratic formula. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity inside the radical, $b^2 - 4ac$, is called the *discriminant.* It is easy to see that we have two real roots if the discriminant is positive, one real root if the discriminant is 0 and no real roots if the discriminant is negative.

1. Find all tangent lines to the parabola $y = x^2$ that pass through the point $(0, -2)$.

2. Consider the parabola $y = x^2$, and a point (c, d) which may or may not lie on the parabola. We will determine how many tangent lines to the parabola pass through (c, d). The exercises below answer this question and allow you to relate the number of tangent lines to the location of the point.

   (a) For each of the points below, make a sketch which shows the parabola given by $y = x^2$ and all tangent line(s) to this parabola which pass through the specified point.

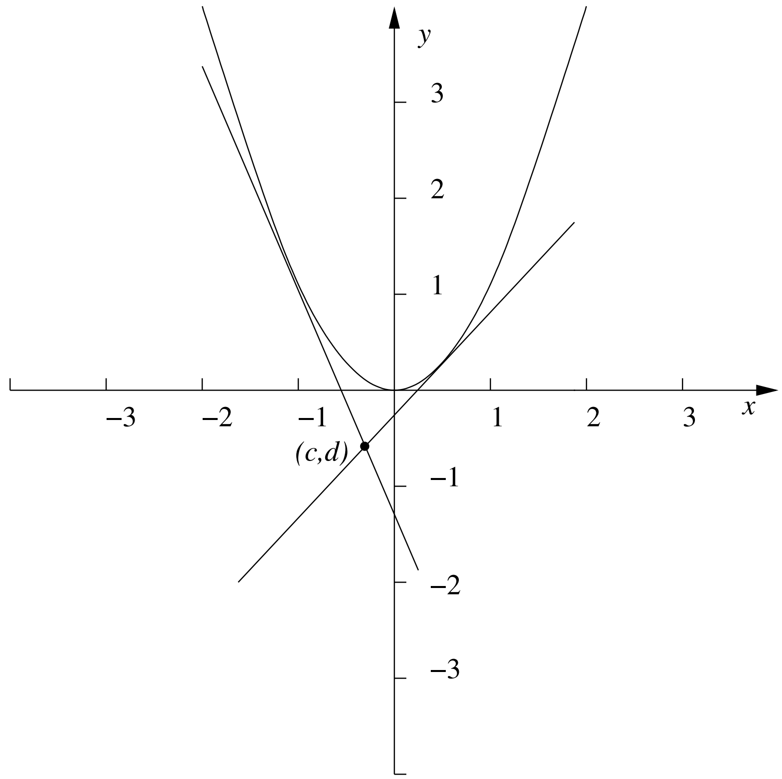
      i. $(1, -2)$
      ii. $(1, 1)$
      iii. $(0, 1)$.

   (b) Make a conjecture as to how many tangent lines of the parabola pass through a given point (c, d). How does the answer depend on the point (c, d)?

   (c) Write the equation of the tangent line to the parabola $y = x^2$ at (a, a^2).

   (d) If we require the tangent line in part c) to pass through point (c, d), we obtain an equation for a. Write out this equation. Solving this equation will give the x-coordinate of the point where the tangent line meets the parabola.

   (e) Give conditions on c and d which tell us that we have exactly 0, 1 or 2 tangent lines through (c, d). Interpret your answers geometrically. What do these conditions tell us about the location of the point (c, d)?



February 4, 2007