Calculus II *Fall 2003*
MA114:007-009 *Russell Brown*

The second exam will be given on Tuesday evening 21 October in CB 122. The exam will cover the sections 7.1-7.6 and 7.8-7.9.
Students should review the trigonometric identities from the first exam. Please know the entries 1-17 on the integral table in the back of Stewart.

1. Integration by parts. Students should be able to recognize integrals which can be evaluated using integration by parts.
2. Trigonometric integrals. Know the strategy for evaluating $\int \sin^k(x)\cos^j(x)\,dx$ when j and k are non-negative integers.
3. Trigonometric substitution. Use of the substitution $x = \sin u$ to evaluate integrals of the form $\int x^k(\sqrt{a^2 - x^2})^j\,dx$.
4. Integration of rational functions by partial fractions. You do not need to be able to use the reduction formula to integrate functions where the denominator is an irreducible quadratic raised to a power larger than one.
5. Rationalizing substitutions. Substituting $u = \sqrt[n]{ax + b}$.
6. Approximate integration. Memorize Simpson's rule and the trapezoid rule. Be able to use the error estimates to estimate the accurracy of the integration. You do not need to memorize the error estimates.
7. Evaluate improper integrals. Be able to state and use the comparison theorem.

Sample exams

1. Compute 5 of the 6 indefinite integrals below. Write here ______ the letter of the integral that is not to be graded. If you do not specify the an integral which is not to be graded, we will take the five lowest scores.

   (a) $\displaystyle\int \sin^2 x\,dx$

   (b) $\displaystyle\int \sin^3 x\,dx$

   (c) $\displaystyle\int \frac{1}{(4 - x^2)^{3/2}}\,dx$

   (d) $\displaystyle\int \frac{x}{x^2 + 2x + 2}\,dx$

   (e) $\displaystyle\int x\cos(2x)\,dx$

(f) $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$

2. Find the form of the partial fraction decomposition for the following rational functions. DO NOT SOLVE FOR THE CONSTANTS.

   (a) $\displaystyle\frac{x}{(x^2-4)(x^2+4)}$

   (b) $\displaystyle\frac{x^2-43x}{(x+1)^3(x+2)(x^2-4)}$

   (c) $\displaystyle\frac{x+1}{x^2(x^2+1)^2(x^2+2x+1)}$

3. The trapezoid rule T_n and Simpson's rule S_n for approximating the integral $\int_a^b f(x)\,dx$ are

$$\begin{aligned} T_n &= \frac{h}{2}(f(x_0)+2f(x_1)+\ldots+2f(x_{n-1})+f(x_n)) \\ S_n &= \frac{h}{3}(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots+4f(x_{n-1})+f(x_n)) \end{aligned}$$

   The errors satisfy

$$|E_T| \le \frac{M_2(b-a)^3}{12n^2} \quad \text{and} \quad |E_S| \le \frac{M_4(b-a)^5}{180n^4}$$

   where M_k is a number which satisfies $|f^{(k)}(x)| \le M_k$ for all x with $a \le x \le b$.

   (a) Use the trapezoid rule and Simpson's rule with $n=4$ to approximate the integral

$$\int_3^6 \sin(2x)\,dx.$$

   Give your answers correctly rounded to four decimal places.

   (b) Find n so that the error in the trapezoid rule is at most 10^{-4},

$$\left|\int_3^6 \sin(2x)\,dx - T_n\right| \le 10^{-4}.$$

   (c) Find n so that the error in Simpson's rule is at most 10^{-4},

$$\left|\int_3^6 \sin(2x)\,dx - S_n\right| \le 10^{-4}.$$

4. (a) State the comparison theorem for improper integrals.

   (b) Use the comparison theorem to determine if the following improper integrals converge.

i. $\int_1^\infty \sin^2(x)e^{-x}\,dx$

ii. $\int_1^\infty \frac{2+\sin x}{x}\,dx$

5. There are thirty-two teams in the 2002 FIFA Copa del Mundo.

   Thus, it is interesting to know the value of the integral

$$\int_0^\infty x^{32}e^{-x}\,dx.$$

   Suppose that we know that

$$\int_0^\infty x^{31}e^{-x}\,dx = A.$$

   Find a simple expression involving A which gives the value of the integral

$$\int_0^\infty x^{32}e^{-x}\,dx.$$

   It is known that $A = 8,222,838,654,177,922,817,725,562,880,000,000$, but I doubt this is very helpful.

1. Compute the following integrals, if possible. If an improper integral diverges, say so.

   (a) $\int_0^{\pi/2} \sin^3 x\,dx$

   (b) $\int \sin^2 x\,dx$

   (c) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

   (d) $\int_0^1 xe^x\,dx$

   (e) $\int \frac{x^2}{x^2+4}\,dx$

   (f) $\int \frac{1}{x^2+x^3}\,dx$

   (g) $\int \frac{1}{x^2+x}\,dx$

   (h) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

   (i) $\int_0^{\pi/2} \tan x\,dx$

   (j) $\int_0^\infty \frac{1}{1+x^2}\,dx$

2. Which of the integrals a), c), h), i) and j) are improper? For each of these integrals, give the point or points where it is improper.

3. (a) Give the definition of a rational function. Give an example of a rational function.
   (b) Give the definition of a proper rational function. Give examples of a proper rational function and an improper rational function.

4. (a) Find the anti-derivative

$$\int \sqrt{1-x^2}\,dx.$$

   (b) Use your answer to compute

$$\int_0^1 \sqrt{1-x^2}\,dx,$$

   (c) The integral in part b) represents the area of a familiar region. Use a geometric argument to give the area and check your answer to part b).

`There were additional problems on the original exam, however these problems are omitted from the review sheet because they examine material that was not covered in this course.`

October 12, 2003