Calculus II *Fall 2003*
MA114:007-009 *Russell Brown*

The third exam will take place 7:30-9:30 on Tuesday, 18 November 2003 in CB122. The exam will cover §8.1, §8.2 and §10.1–10.8.
The topics to be examined are:

1. Separable differential equations and applications.
2. Arc-length of graphs.
3. Sequences. Monotonic sequences.
4. Convergent series. Telescoping series, geometric series.
5. Integral test and its use in estimating series.
6. Comparison test and limit comparison test.
7. Alternating series and the estimate for the error.
8. Absolute convergence, conditional convergence.
9. Ratio test.
10. Convergence of power series, radius of convergence.

Review assignment: Chapter 8, page 541, #1–8, 9a, 25, 26, 27. Chapter 10, page 675–676. #1–9, 11, 12, 14–60.

Sample exam questions

1. Find the following limits.
   (a) $\lim_{n\to\infty} \frac{2n^2-1}{3n^2+987}$
   (b) $\lim_{n\to\infty} \frac{n!}{(n+1)!}$
2. Find the sum of each of the following series.
   (a) $\frac{1}{3\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{3\cdot 8}+\frac{1}{3\cdot 16}+\ldots+\frac{1}{3\cdot 2^n}+\ldots$
   (b) $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

3. Determine if each of the following series is absolutely convergent, conditionally convergent or divergent. Describe briefly how you test the series for convergence or divergence.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

   (c) $\sum_{n=1}^{\infty} (-3)^n$

4. Let

$$s = \sum_{n=1}^{\infty} \frac{1}{n^5} \quad \text{and} \quad s_N = \sum_{n=1}^{N} \frac{1}{n^5}.$$

   Use the integral test to find a value of N so that $|s - s_N| < 10^{-5}$.

5. Find the radius and interval of convergence for the following power series.

   (a) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$.

   (b) $\sum_{n=1}^{\infty} \frac{x^n}{(2n+1)}$

   (c) $\sum_{n=1}^{\infty} n(2x-5)^n$

6. (a) State the alternating series estimation theorem.

   (b) Find a value of N so that

$$\left|\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} - \sum_{n=0}^{N} \frac{(-1)^n}{n!}\right| \leq 10^{-3}.$$

1. Find the limits of the following sequences.

   (a) $\lim_{n\to\infty} \frac{n^2+1}{2n^2+n}$

   (b) $\lim_{n\to\infty} \frac{1}{1+2^{-n}}$

   (c) $\lim_{n\to\infty} \frac{1}{1+2^n}$

2. Suppose that a sequence $a_1, a_2, \ldots$ is defined by

$$a_{n+1} = 4 - \frac{1}{a_n} \quad \text{and} \quad a_1 = 1.$$

It can be shown that this sequence is bounded and increasing. Assuming that the sequence is bounded and increasing, explain why the sequence has a limit and compute the exact value of the limit

$$\lim_{n\to\infty} a_n.$$

(You may check your answer by finding an approximate value for the limit using your calculator.)

3. Determine if each of the following series converges and, for the convergent series, compute the sum of the series.

(a) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(b) $\sum_{n=4}^{\infty} 2^{-n}$

(c) $\sum_{n=0}^{\infty} 3^n$

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

4. (a) State the integral test for convergence of a series.

(b) What N is needed so that

$$\left|\sum_{n=1}^{\infty} \frac{1}{n^5} - \sum_{n=1}^{N} \frac{1}{n^5}\right| < 10^{-5}?$$

Do not evaluate the partial sum.

5. For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

converge.